January 1976
Report No. Env. E. 53-76-1

APPLICATION OF A SLUDGE GRAVITY THICKENING MODEL

Leo T. Mulcahy
Peter Kos
Donald Dean Adrian
Tsuan Hua Feng

Report for Division of Water Pollution Control
Massachusetts Water Resources Commission

Contract Number 73-07(3)

ENVIRONMENTAL ENGINEERING
DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF MASSACHUSETTS
AMHERST, MASSACHUSETTS

APPLICATION OF A SLUDGE GRAVITY THICKENING MODEL

By

Leo T. Mulcahy[1]

Peter Kos[2]

Donald Dean Adrian[3]

Tsuan Hua Feng[3]

Division of Water Pollution Control
Massachusetts Water Resources Commission
Contract Number MDWPC 73-07(3)

Environmental Engineering Program
Department of Civil Engineering
University of Massachusetts
Amherst, Massachusetts 01002

January 1976

---

[1] Teaching Associate, Univ. of Massachusetts/Amherst

[2] Environmental Technologist, Dorr-Oliver Inc., Stamford, Connecticut 06904

[3] Professor of Civil Engineering, Univ. of Massachusetts/Amherst

TABLE OF CONTENTS

ABSTRACT

Kos (3) has recently advanced a mathematical model that describes the continuous gravity thickening process. The model is based on an analysis of transport phenomena within the thickening zone.

The objective of this report is to present an outline of this new model and describe how it can be used as a rational basis for the design of both gravity thickeners and clarifiers.

LIST OF FIGURES

LIST OF TABLES

SECTION 1.

INTRODUCTION

SECTION I - INTRODUCTION

Disposal of sludges produced by water and wastewater treatment processes is a topic of major engineering concern. It has been reported that the effectiveness of sludge handling is the single most important variable affecting the cost of wastewater treatment plant operation and maintenance (1).

Gravity thickening has long been recognized as an effective unit operation for sludge volume reduction. Such volume reduction greatly enhances the cost effectiveness of subsequent sludge handling techniques.

Thickening is also an important phenomenon when considering the performance of secondary clarifiers in the activated sludge process. Secondary clarifiers must accomplish a two fold function: production of a clarified overflow and production of an adequately concentrated underflow. Both the clarification and thickening function must be considered in design if the clarifier is to satisfactorily accomplish both of its tasks.

Inattention to the thickening function can lead to loadings that exceed the clarifiers ability to transmit solids to the tank bottom. Solids overloadings of sufficient magnitude and duration can cause the height of the sludge layer to approach the overflow weir causing a drastic deterioration in effluent quality.

In addition, the success of the activated sludge process is dependent on an ability to maintain a high concentration of active microorganisms in the aeration unit. In practice this high concentration of microorganisms is maintained by recycling biologically active solids from the secondary clarifier. Efficient recycle of solids can be realized only if an adequate degree of thickening is accomplished in the clarifier. Inadequate thickening in the secondary clarifier can lead to disruption of the entire process by limiting the concentration of active biomass in the aeration unit.

In spite of the engineering importance of the thickening process, designers have been forced to rely on experience and empirical analyses when sizing thickeners or clarifiers. Bacon and Dalton highlighted the need for a rational approach to design when they called sludge thickening "the largest unsolved research and development problem" in the wastewater treatment field (2).

Kos (3) has recently advanced a mathematical model that describes the gravity thickening process. The model is based on an analysis of transport phenomena within the sludge layer during thickening. By using this new mathematical model to predict thickening behavior, design of thickeners as well as design for the thickening function of clarifiers can now be conducted on a wholly rational basis. The model can also be useful in predicting the impact of operational variations on the performance of existing thickeners and clarifiers.

This report presents an outline of the theoretical basis of Kos' model and details the equipment and experimental procedure necessary for its implementation.

PROBLEM BACKGROUND

A review of recent literature on gravity thickening indicates a gradual shift in the approach used by researchers to view the thickening process (4,5,6,7). An increasing number of investigators have come to realize that any rigorous examination of the thickening process must be based on a detailed analysis of the filtration and deformation processes occurring within the sludge layer during thickening. Basic to such an analysis is a thorough understanding of the physical nature of the sludge layer.

SLUDGES AND FLOCCULENT SUSPENSIONS

Sludges generated in water and wastewater treatment plants are mixtures of various organic and inorganic materials in the solid, liquid and gaseous states. The composition of each sludge is dependent on both the origin of the treated water and the type of technology used in treatment. Although sludges differ in specific composition their solid phase is principally flocculent in nature. The characteristic feature of flocculent suspensions is that individual solid particles are unstable and tend to aggregate into loosely bound masses of particles known as flocs.

Under steady-state conditions, flocculent suspensions form three zones within a continuous gravity thickener (see Figure 1). Micro-flocs of sludge introduced to the unit continue to agglomerate while settling through the sedimentation zone. As flocs settle on and become part of

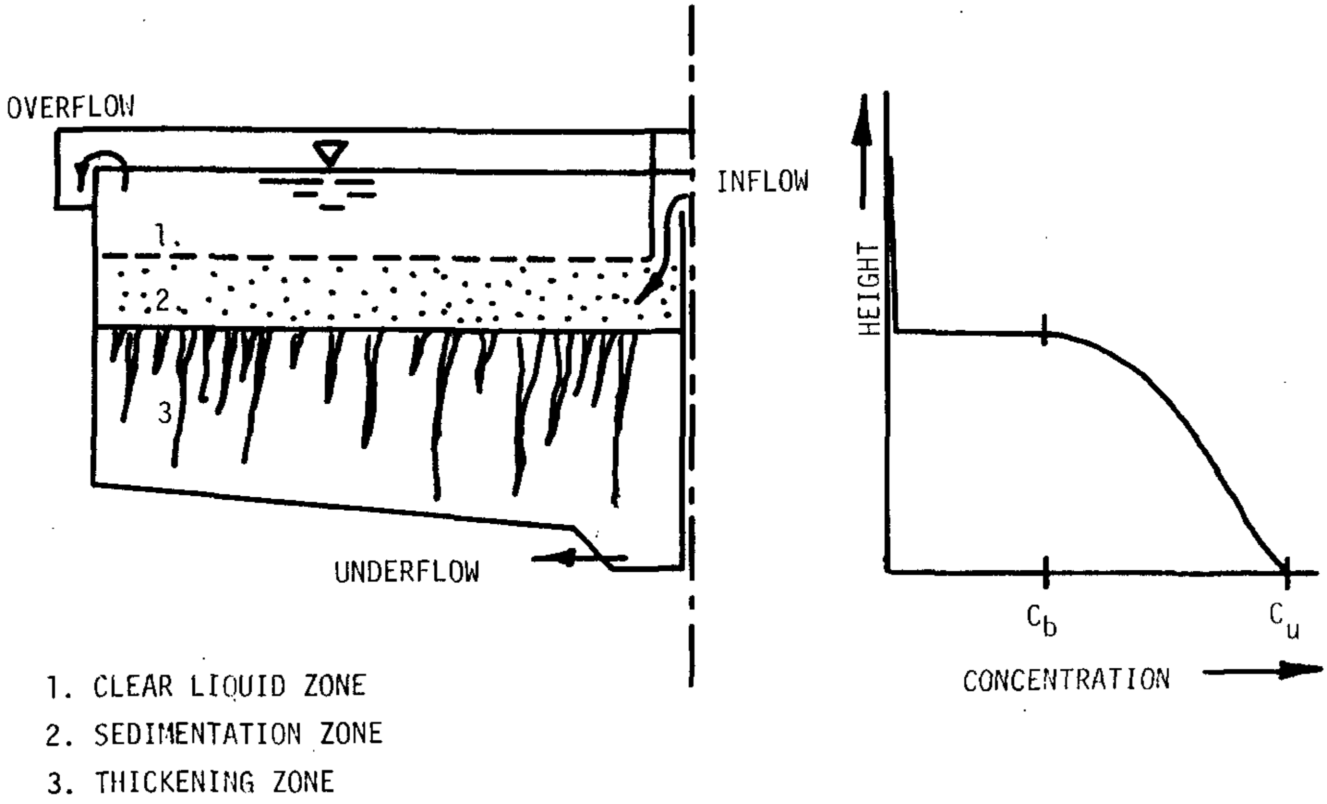


FIGURE 1. TYPICAL CONCENTRATION PROFILE IN A CONTINUOUS GRAVITY THICKENER

the thickening zone they lose their individual character. They become part of the continuous porous matrix of solids that is known as the thickening zone.

To mathematically model the thickening process one must be able to describe mathematically the basic physical phenomena which take place within the solids matrix. At the foundation of any such mathematical description are system conservation equations. Conservation equations are mathematical relationships which state that the rate of accumulation of any conserved entity within a system is equal to the rate of addition of that entity to the system less the rate at which the entity is removed from the system. Some examples of entities for which conservation equations can be written are: mass, momentum, moment of momentum, various forms of energy, entropy and electrical charge.

While providing the foundation for the mathematical modeling of physical systems, conservation equations alone generally cannot provide enough information to permit an adequate prediction of system behavior. In order to fully describe the behavior of a system it is often necessary that constitutive relationships be incorporated into the model. In constitutive theory the real (observed) behavior of a material is approximated by mathematical postulates, the constitutive equations. These equations connect or restrict some of the variables introduced in the formation of system conservation equations.

The following section of this report presents a mathematical model of the gravity thickening process. The model has been formulated by applying conservation and constitutive theory to the deformable, porous matrix of solids that is the sludge layer.

SECTION 2.

DEVELOPMENT OF THE MODEL

SECTION II - DEVELOPMENT OF THE MODEL

CONSERVATION EQUATIONS

In modeling the gravity thickening process the conserved entities of interest are mass and momentum. The mass and momentum balances presented here have been derived by Raats (8).

Conservation of Mass

A mass balance on the solid phase of the thickening zone has the form:

$$\partial\rho_s/\partial t + \partial(\rho_s v_s)/\partial x = 0 \tag{1}$$

where

ρ_s = bulk density of the solid phase*

t = independent variable, time

v_s = spatial (or Eulerian) coordinate within the thickening zone.

A mass balance on the liquid phase within the thickening zone has the form:

$$\partial\rho_\ell/\partial t + \partial(\rho_\ell v_\ell)/\partial x = 0 \tag{2}$$

---

*For continuous gravity thickening, ρ_s can be defined as the mass of solids per unit volume of sample.

where

v_ℓ = velocity of the liquid phase at point x

ρ_ℓ = bulk density of the liquid phase i.e. mass liquid/unit volume of sample.

For a continuous thickener, operated at steady state, Equations 1 and 2 may be integrated to yield, respectively, Equations 3 and 4.

$$\rho_s v_s = \text{constant} = G_s \tag{3}$$

where

G_s = mass flux of the solid phase

and

$$\rho_\ell v_\ell = \text{constant} = G_\ell \tag{4}$$

where

G_ℓ = mass flux of the liquid phase.

Introducing the relationship

$$\rho_s/d_s + \rho_\ell/d_\ell = 1 \tag{5}$$

where

$d_s = \dfrac{\text{mass solid}}{\text{unit volume of solid}}$; density of solid phase

$d_\ell = \dfrac{\text{mass liquid}}{\text{unit volume of liquid}}$; density of liquid phase permits Equations 3 and 4 to be expressed in terms of flows and concentrations as follows: (see Figure 2 for additional description of nomenclature)

$$\rho_s v_s = G_s = Q_u c_u / A_u \qquad (6)$$

$$\rho_\ell v_\ell = G_\ell = (A_u c_u / A) d_\ell \left[\frac{1}{c_u} - \frac{1}{d_s}\right] \qquad (7)$$

where

Q_u = volumetric underflow rate

c_u = underflow suspended solids concentration;
$c = \rho_s \times 10^3$

A = surface area of thickener.

Superficial velocity $(v_s - v_\ell)n$ may also be expressed in terms of flows and concentrations:

$$(v_s - v_\ell)n = (Q_u c_u / A) \left[\frac{1}{c(x)} - \frac{1}{c_u}\right] \qquad (8)$$

where

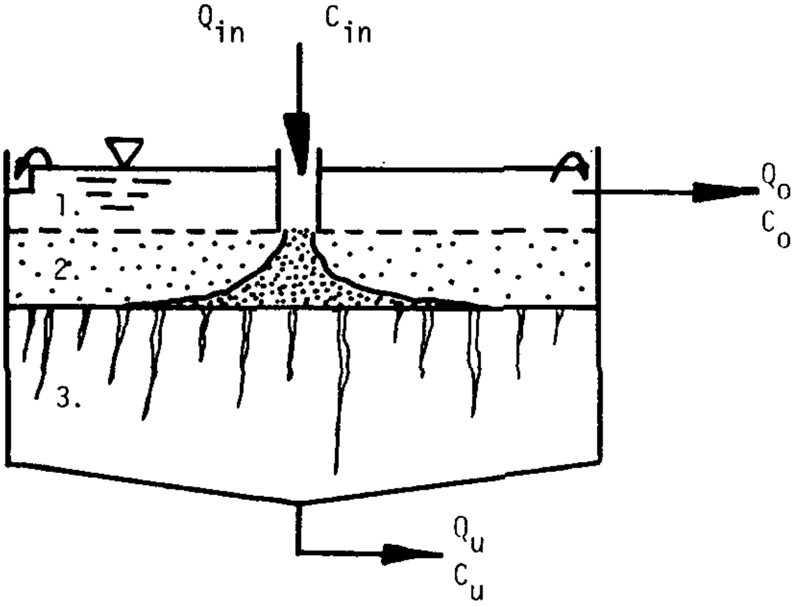
$c(x)$ = suspended solids concentration at level x

n = porosity ($n = \rho_\ell / d_\ell$).

A complete derivation of Equation 8 is given in Appendix III of Kos (3).

Conservation of momentum

The form of the momentum (or force) balance for the solid phase is dependent on the physical model used to describe the porous medium. Based



1. CLEAR LIQUID ZONE
2. SEDIMENTATION ZONE
3. THICKENING ZONE

FIGURE 2. SCHEMATIC OF A CONTINUOUS GRAVITY THICKENER

on earlier research (9) Kos chose to view the thickening zone as a granular porous medium with point contacts between solid particles. A force balance on the solid phase then has the form:

$$(-\rho_s)\partial v_s/\partial t - \partial\sigma/\partial x + \rho_s g_x - (1-n)\ \partial p/\partial x + F(v_\ell - v_s) = 0 \qquad (9)$$

where

σ = effective (or interparticle) pressure
g_x = gravity acceleration in the x direction
n = porosity
p = liquid phase pressure
F = resistivity.

The first term in Equation 9 represents the net influx of momentum by solids flow (inertial force). The second term represents the functional relationship between interparticle pressure force and thickening zone depth. The third term accounts for the gravity force acting on the solid phase. The fourth term represents the change with depth of the force exerted by the liquid phase on the solid phase (i.e. the buoyant force). The fifth term accounts for the momentum transfer between the liquid phase and the solids matrix by viscous effects.

Analogously the liquid force balance has the form:

$$(-\rho_\ell)\partial v_\ell/\partial t - n\partial p/\partial x + \rho_\ell g_x - F(v_\ell - v_s) = 0 \qquad (10)$$

which is a balance of the inertial force (first term), change in liquid phase pressure force with depth (second term), gravitational force (third term) and viscous drag force (fourth term).

Because the movement of both solid and liquid phases during thickening is very slow, the magnitudes of the inertial (first) terms in Equations 9 and 10 are small in comparison to the other terms in these equations. Therefore inertial terms will be neglected in subsequent steps involving force balances.

CONSTITUTIVE EQUATIONS

Constitutive theory is used to describe mathematically the transfer of force within the thickening zone from the liquid to the solid phase. This transfer of force is represented in the force balances (Equations 9 and 10) as:

$$F(v_\ell - v_s)$$

The resistivity to flow, F, is dependent on the physical properties of both the solid and liquid phases. In order to separate the effects of each phase on momentum transfer, resistivity to flow is redefined:

$$F = \mu n^2/k \tag{11}$$

where

μ = viscosity of the liquid phase

n = porosity; $n = \rho_\ell/d_\ell$

k = intrinsic conductivity of the solid phase*.

Equations 10 and 11 can be combined to yield a relationship to Darcy's law.

$$dp/dx = d_\ell g_x + \frac{\mu}{k}(v_\ell - v_\ell)n \tag{12}$$

The viscous transfer of momentum between phases during thickening can be predicted if the intrinsic conductivity of the sludge matrix can

---

*Kos(3) shows k to be a function of both local solids concentration and superficial velocity.

be determined experimentally. The intrinsic conductivity of a sludge is evaluated by determining the relationship among measured values of liquid phase pressure drop, superficial velocity and suspended solids concentration. Details of the evaluation are given elsewhere in this report.

Another constitutive relationship is used to provide a mathemathical description of the compressive behavior of a sludge during thickening. For this purpose a stress-strain type relationship is introduced that relates consolidation of the sludge matrix (strain) to stress transmitted through interparticle contacts.

Constitutive equations of this type are well known in the field of soil mechanics where the constant used to define the stress-strain relationship for a particular soil is known as the coefficient of linear compressibility and symbolized by the letter a. It has been observed that the compressive behavior of sludges differs from the compressive behavior of most soils in that for sludges the modulus of linear compressibility is also dependent on local solids concentration. In spite of this, it is still convenient to use the coefficient of linear compressibility concept to describe the consolidation of a sludge. The concentration dependency simply complicates the task of experimentally determining the coefficient of linear compressibility.

The coefficient of linear compressibility has been defined as:

$$a = d\rho_s/d\sigma \tag{13}$$

where

a = modulus of linear compressibility; $a = a(\rho_s)$

ρ_s = bulk density of the solid phase

σ = effective (or interparticle) pressure

Or, substituting the more easily measured quantity, c (suspended solids concentration), Equation 13 is redefined:

$$\bar{a} = dc/d\sigma \tag{13a}$$

where

$\bar{a}$ = modulus of linear compressibility; $\bar{a} = \bar{a}(c)$

c = suspended solids concentration.

Thus the consolidative behavior of the sludge during thickening can be predicted if the relationship between measured values of suspended solids concentration and effective pressure within the sludge layer can be described mathematically. Details of this mathematical description of consolidation are given elsewhere in this report.

SUMMARY OF THE MODEL

Equations 9 and 10 can be combined to yield an overall force balance on the two phase system (i.e. on the sludge layer during thickening). This overall force balance can be rearranged into the following form:

$$d\sigma/dx + dp/dx = g_x(\rho_\ell + \rho_s) \tag{14}$$

which upon integration becomes:

$$\sigma(x) + p(x) = g_x \int_0^x (\rho_\ell + \rho_s)dx \, . \tag{15}$$

The term $(\rho_s+\rho_\ell)$ in equations 14 and 15 represents the mass density of the suspension as a whole, i.e., $d_{s\ell} = (\rho_s+\rho_\ell)$. This quantity is a function of suspended solids concentration, i.e., $d_{s\ell} = d_{s\ell}(c)$. Equation 14 can be rewritten:

$$\frac{d\sigma_T}{dx} = d\sigma/dx + dp/dx = g_x d_{s\ell} \tag{14a}$$

where

σ_T = total pressure

$d_{s\ell} = d_\ell + (1 - d_\ell/d_s)c$.

Equation 15 states that the total pressure (quantity to the right of the equality) is equal to the sum of the effective pressure (force carried by the solid phase) and the liquid or piezometric pressure (force carried by the liquid phase).

At this point all equations that were used to model the steady state continuous gravity thickening process have been presented. These equations are summarized in Table 1.

In addition, these equations can be used to develop a single differential equation for concentration change as a function of thickening zone depth,

$$\frac{dc(x)}{dx} = \bar{a}(x)[g_x d_{s\ell}(x) - d_\ell g_x - \frac{\mu}{k(x)} G_s(\frac{1}{c(x)}) - \frac{1}{c_u})]. \qquad (16)$$

If, however, information on the distribution of liquid and effective pressure within the thickening zone is also desired the individual equations (8, 12, 13a, 14a)must be applied.

To predict the thickening behavior of a particular sludge using the equations summarized in Table 1 the filtration and consolidation characteristics of that sludge must be determined experimentally. This is done by operating a laboratory continuous thickener at a steady state (corresponding to a given set of operating conditions) and measuring liquid phase and effective pressure profiles as well as the suspended solids concentration profile. The information provided by these profiles is then used in the equations of Table 1 to calculate the filtration and consolidation characteristic of the sludge. Having experimentally determined these characteristics for a sludge the equations of Table 1 can then be used to predict the pressure and concentration profiles that correspond to any set of operating conditions.

Table 1. Summarization of Equations for the Description of Steady-State Continuous Gravity Thickening

---

$$(v_s - v_\ell)\, n_\ell = \frac{Q_u c_u}{A} \left[\frac{1}{c(x)} - \frac{1}{c_u}\right] \tag{8}$$

$$\frac{dp}{dx} = d_\ell\, g_x + \frac{\mu}{k(x)} (v_s - v_\ell)\ \ n \tag{12}$$

$$\frac{d\sigma}{dx} = \frac{d\ \sigma_T}{dx} - \frac{dp}{dx} = d_\ell\ \ g_x + g_x \left(1 - \frac{d_\ell}{d_s}\right) C(x) - \frac{dp}{dx} \tag{14a}$$

$$a = \frac{dc}{d\sigma} \tag{13a}$$

$$\frac{dc}{dx} = \bar{a}(x) \left[g_x\ d_{s\ell}(x) - d_\ell\ \ g_x - \frac{\mu}{k(x)}\ \ G_s \left(\frac{1}{c(x)} - \frac{1}{c_u}\right)\right] \tag{16}$$

SECTION 3.

EXPERIMENTAL EQUIPMENT AND PROCEDURES

SECTION III - EXPERIMENTAL EQUIPMENT AND PROCEDURES

Two physical characteristics of a sludge must be determined experimentally before the mathematical model presented in this report can be used to predict the thickening behavior of the sludge.

In order to determine these physical characteristics detailed information on the suspended solids concentration and pressure (interparticle and liquid phase) distributions within a gravity thickener under various operational conditions must be obtained. The following is a brief description of the laboratory thickener used by Kos to obtain such information.

THE LABORATORY GRAVITY THICKENER

The gravity thickener used by Kos (3) was constructed from sections of 11 1/2 in (29 cm) I.D. plexiglass column such that the overall column depth was 8 1/2 ft (259 cm). The thickener was equipped with feed well, overflow trough, rake and stirring rods*.

A 55 gallon steel drum was used as a feed tank. To achieve uniform feed concentration the feed tank was fitted with a large propeller-type stirrer driven by a 1/3 horsepower electric motor. The rate at which sludge was pumped from the feed tank to the feed well of the thickener was controlled by a Sigmamotor pump. The rate at which concentrated

---

*Kos examined the effect of stirring on both batch and continuous thickening and found that it had a detrimental effect on continuous thickening. Therefore stirring is not recommended as part of the experimental procedure outlined in this report. For additional details on the effect of stirring on the thickening process the reader is referred to Kos (3).

sludge was withdrawn from the thickener (underflow-rate) was also controlled by a Sigmamotor pump.

Photographs showing the various elements of the gravity thickener are given on pages 20 and 21 of this report.

Total material and equipment costs for this apparatus were approximately $6,000. It is estimated that an additional $5,000 to $6,000 in labor costs would be incurred in assembly of the apparatus.

It was found that close temperature control during operation of the thickener was necessary in order to obtain consistent and reproducible data. Therefore all tests described in this report were carried out in a laboratory equipped with a temperature control system. This system is capable of maintaining temperature constant ± 0.5°C.

As mentioned earlier, data to be gathered includes detailed information on the suspended solids and pressure distributions within the laboratory thickener under various combinations of operational parameters. The following is a description of the procedure and apparatus used to obtain this information.

SUSPENDED SOLIDS DISTRIBUTION

Suspended solids profile determinations are quite straightforward. The laboratory thickener is fitted with a series of sampling ports (spaced vertically). From each port a known volume of sample is withdrawn and analyzed for suspended solids.

PRESSURE DISTRIBUTIONS

Determination of pressure distributions is more complicated. The

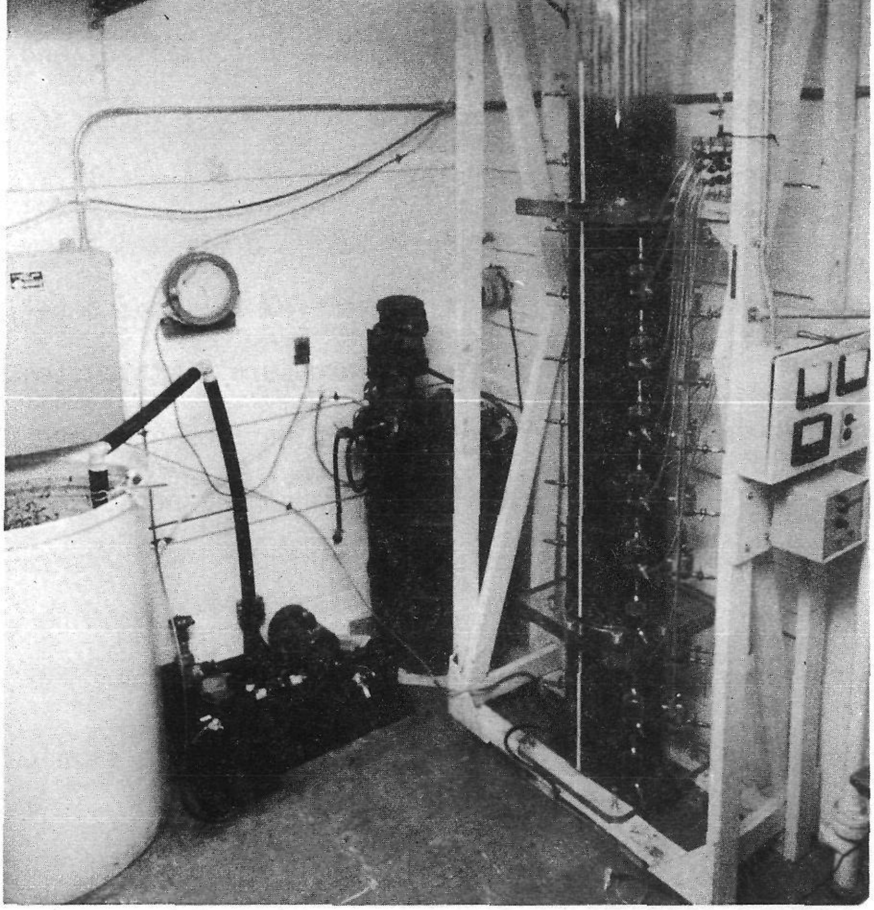


FIGURE 3. GRAVITY THICKENING APPARATUS USED BY KOS (3.).

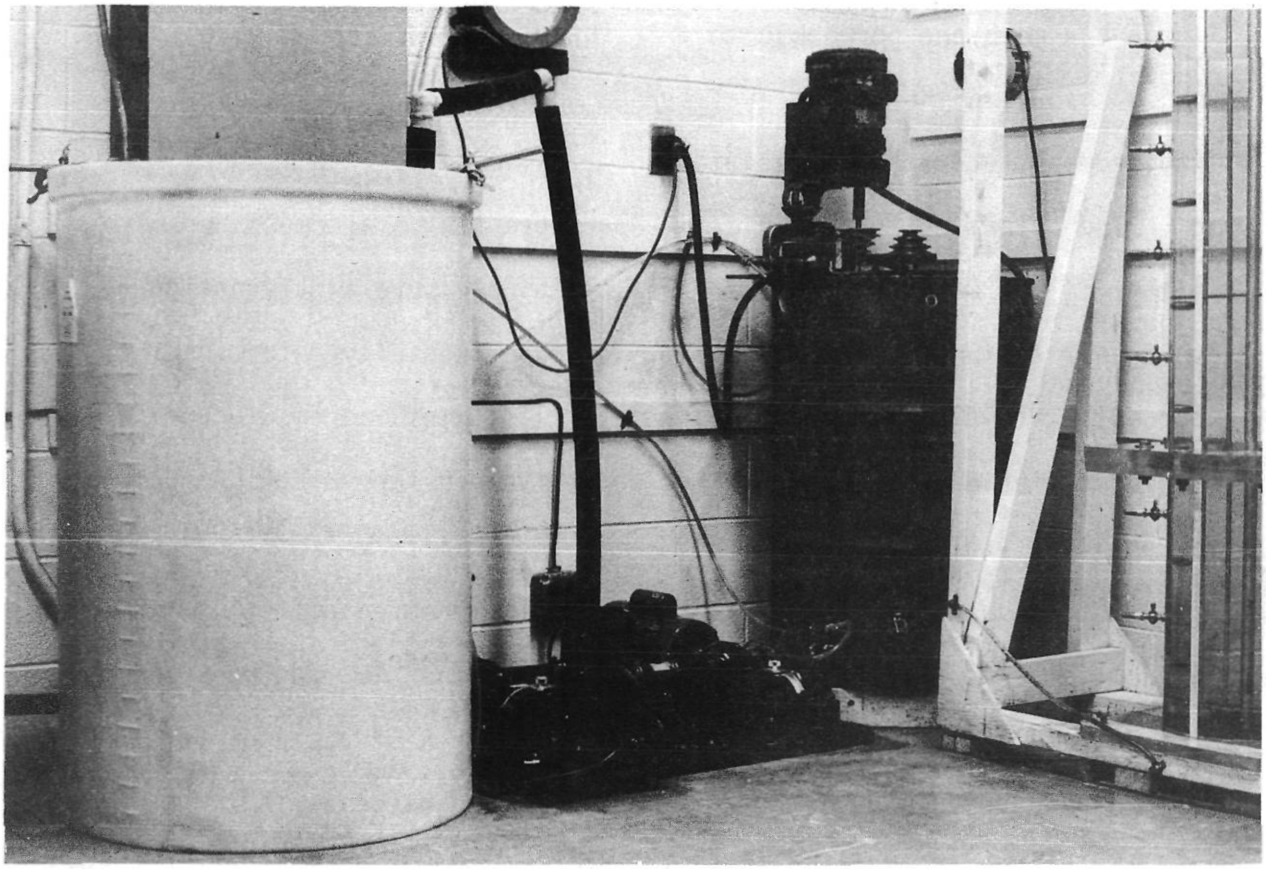


FIGURE 4. APPARATUS USED BY KOS (3.).
LEFT TO RIGHT: SLUDGE STORAGE RESERVOIR, SIGMAMOTOR PUMPS, FEED TANK WITH STIRRER, THICKENER.

mathematical model requires information on both the effective (interparticle) pressure distribution and the liquid phase pressure distribution.

Raats (8) describes the liquid phase pressure as corresponding to the pressure in a reservoir of fluid, in contact with the liquid phase of the mixture (the mixture being sludge in this application) through a membrane permeable to the liquid phase but not the solid phase. Thus in the presence of such a membrane, liquid phase pressure can be measured with a piezometer.

The determination of effective pressure can be accomplished by either of two methods. In the first, total pressure as a function of thickener depth is calculated using the experimentally determined concentration distribution. Having calculated total pressure and measured liquid phase pressure the effective pressure distribution can be obtained via equation 14.

In the second method, total pressure is measured directly with a piezometer. As will be explained here this method is made possible by the extremely fragile nature of the solids matrix of a sludge during thickening. Figure 5a represents a continuous thickener that has reached steady state. The liquid phase pressure at depth D corresponds to the height of liquid rise in the piezometer. Total pressure at depth D is the sum of liquid phase pressure and pressure carried by the solids matrix at that depth. If the structure of the solids matrix at depth D could be destroyed the matrix would no longer be capable of supporting the weight of solids above that depth. Support of the solids would necessarily be assumed by viscous drag forces between the solid and liquid phase. Because the total stress (due to both overlying solid

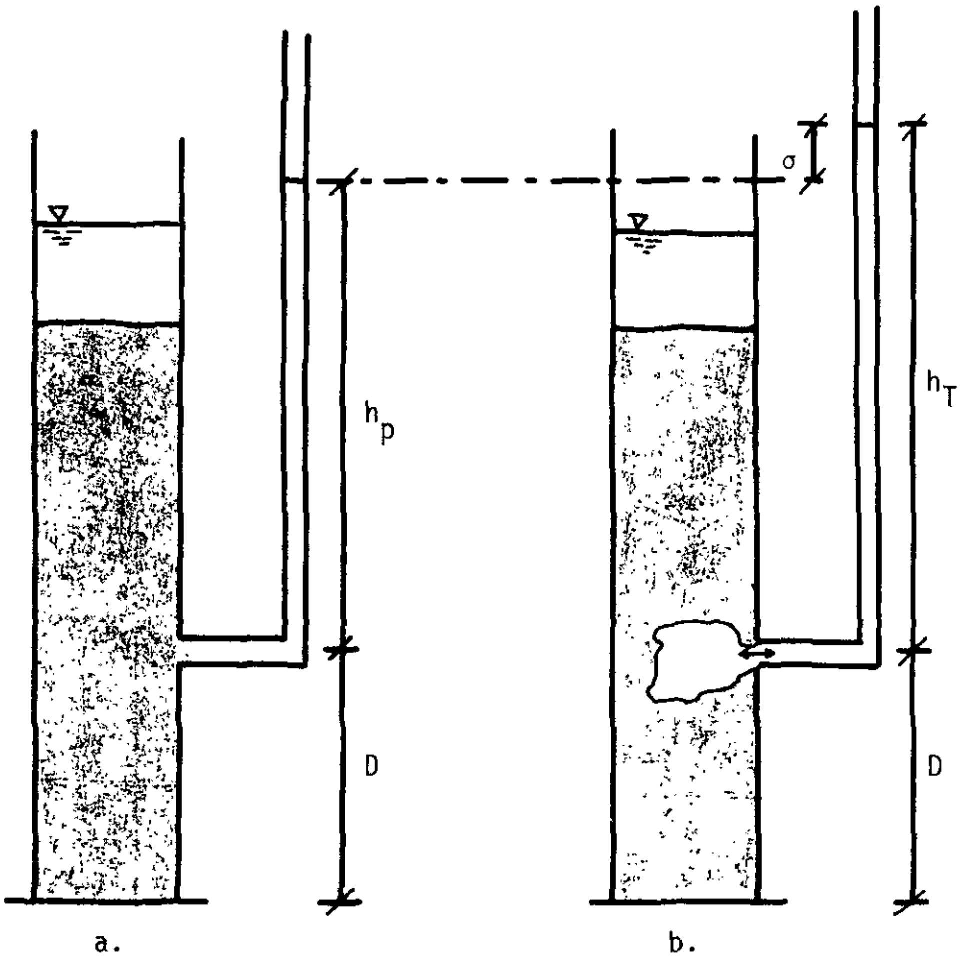


FIGURE 5. SCHEMATIC OF A CONTINUOUS GRAVITY THICKENER

a. BEFORE

b. AFTER

STRUCTURE OF SLUDGE MATRIX IS DESTROYED BY SQUEEZING PIEZOMETER TUBING and liquid phases) is then carried entirely by the liquid phase, the height that the liquid column rises in the piezometer corresponds to the total pressure at depth D.

It was found that the fragile solids matrix could be destroyed by rapidly squeezing the flexible tubing of the piezometer. Figure 5b represents the sludge layer after the structure of the solids matrix has been destroyed. The height h_T in Figure 5b corresponds to the total pressure at depth D. Effective pressure at a given depth is the difference between measured values of liquid phase and total pressure at that depth.

Experimental evidence indicates that comparable results are obtained by either method. The later was chosen as a matter of convenience.

Some of the practical problems encountered in the laboratory with regard to pressure measurements will be discussed here. This is done in an effort to clarify the reasoning behind some of the experimental procedures used.

As Raats (8) has stated, liquid phase pressure can be measured with a piezometer providing the piezometer is separated from the sludge by a membrane that is permeable to the liquid phase while excluding the solid phase. During experimentation, it was found that the sludge itself acted as such a membrane. By drawing a small amount of the sludge into the piezometer a porous "plug" of sludge (which permitted liquid flow while preventing further intrusion of the solid phase) was formed at the thickener - piezometer interface. However,with time,the plug of sludge within the piezometer would itself settle. This necessitates the drawing of an additional volume of sludge into the piezometer to form a new porous "plug" prior to the pressure measurement. This drawing of sludge into the piezometer itself creates additional problems. First, a method must be devised to draw the sludge into the piezometer. Second, sludge solids must not be allowed to build up in the piezometer tubing.

These problems were handled by the use of "separation cells" between the thickener and the piezometer. Details of such cells are shown in Figures 6 and 7.

The porous sludge plug is formed in the fitting which connects the thickener to the separation cell by carefully opening valve A (Figure 7) and allowing a few drops of liquid to pass. This liquid flow draws sludge into the fitting thus forming the plug. The flow also forces the "old"plug out of the fitting. Solids drawn from the fitting settle in the separation cell thereby eliminating the problem of solids accumulation within the piezometer tubing.

Using this arrangement liquid phase and total pressure (as discussed earlier) could theoretically be measured by piezometers attached to separation cells at various depths along the thickener wall. In practice such a simple arrangement lacks the sensitivity necessary to accurately determine the minute difference between total and liquid phase pressures at various depths within the thickener. Great sensitivity is needed because in the gravity thickening of virtually all sludges encountered in the water and wastewater treatment field, pressures transmitted by interparticle contacts (effective pressures) are many orders of magnitude smaller than liquid phase pressures. Refering back to Figure 5, this means that h_T would be only hundredths of an inch greater than h_p. The fact that effective pressure is the

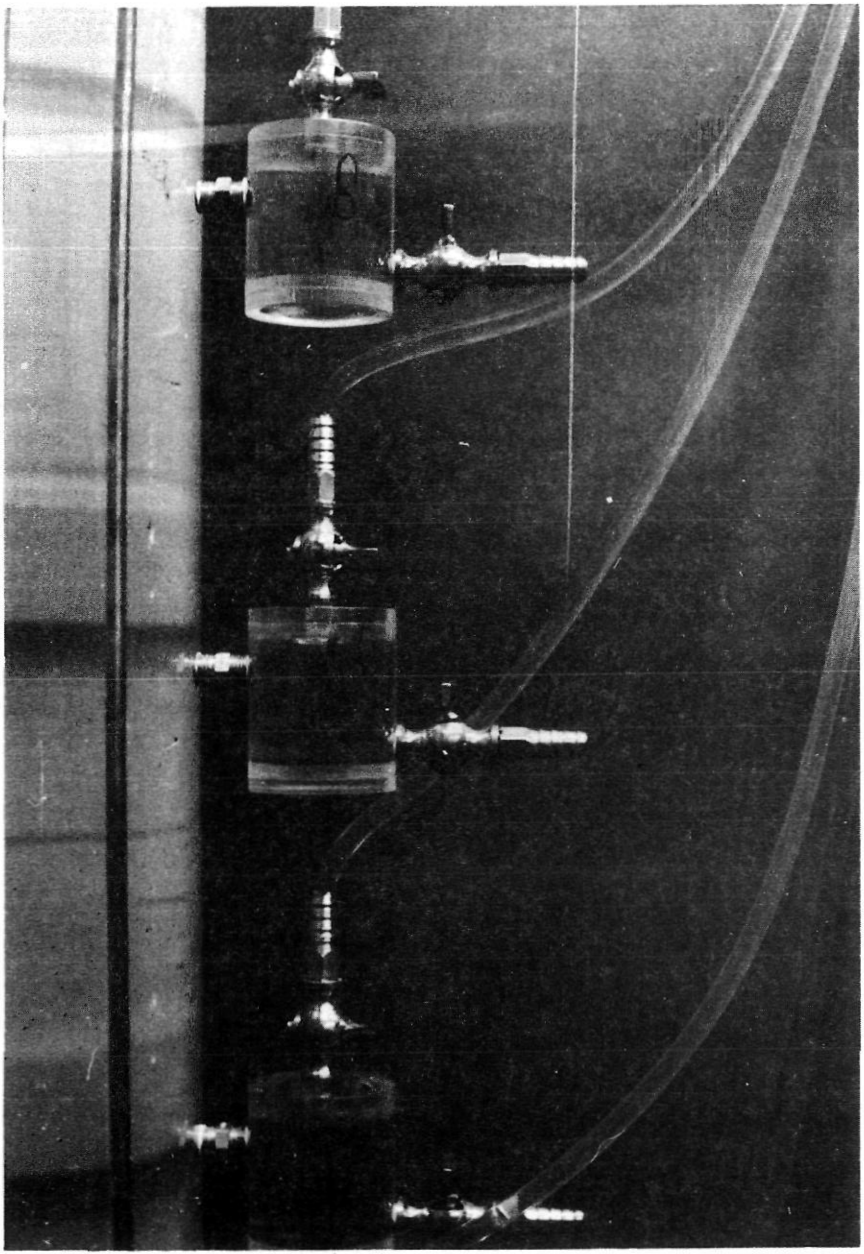


FIGURE 6. SEPARATION CELLS

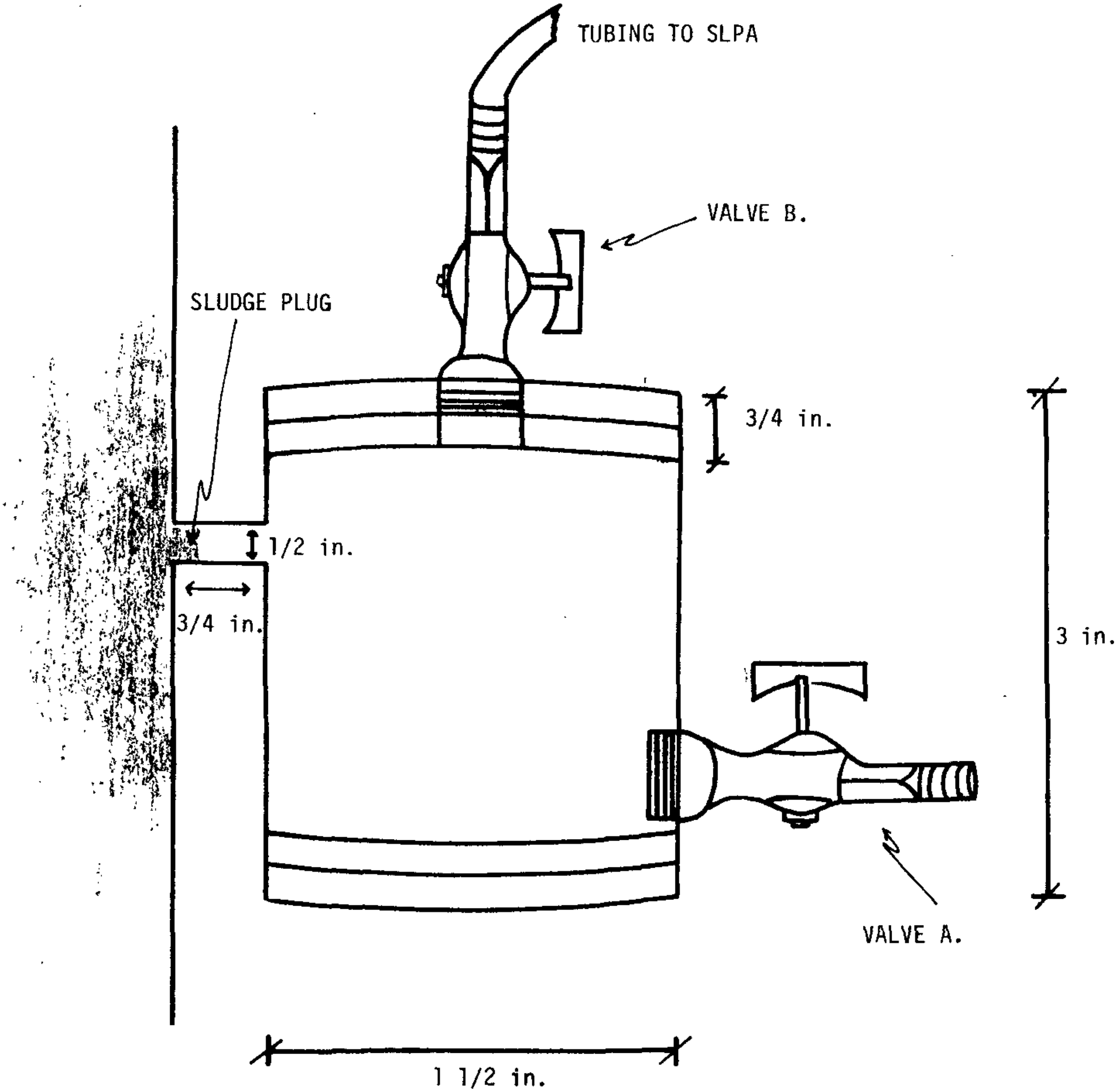


FIGURE 7. SCHEMATIC OF A SEPARATION CELL

single most important parameter governing the thickening process makes its precise measurement critical.

To accurately measure these small pressure differences Kos used a Statham transducer coupled to a Statham transducer readout (hereafter, the combination of these elements will be referred to as the Statham low pressure accessory or SLPA). The SLPA is capable of pressure measurements in the 0 to 0.1 psi range. Figures 8 and 9 are photographs of the SLPA used by Kos.

The datum for pressure measurement is fixed by the height of the transducer. By locating the transducer on a plane a few millimeters below that of the thickener's liquid level hydrostatic head is effectively eliminated from pressure measurements. Thus the SLPA did not have to measure pressures of the relatively large magnitudes that correspond to h_p and h_T of Figure 5 but rather only the portion of those pressures caused by the difference between h_p or h_T and the datum.

When dealing with such small pressures any variation in the height of liquid within the thickener during measurement must be considered. For this reason a reference pressure port was added near the top of the laboratory thickener. By opening the three-way valve (shown in Figure 8) to the reference port, the height of the liquid level above the datum at that time could be accurately measured. The valve position was then changed so that the SLPA measured the pressure in one of the thickeners other pressure ports. By subtracting the reference pressure from the other measured pressure the portion of the measured pressure that corresponds to the hydrostatic head was eliminated. The difference between liquid phase and hydrostatic pressure will be referred to as $\bar{p}$-pressure. The difference between total and hydrostatic pressure will

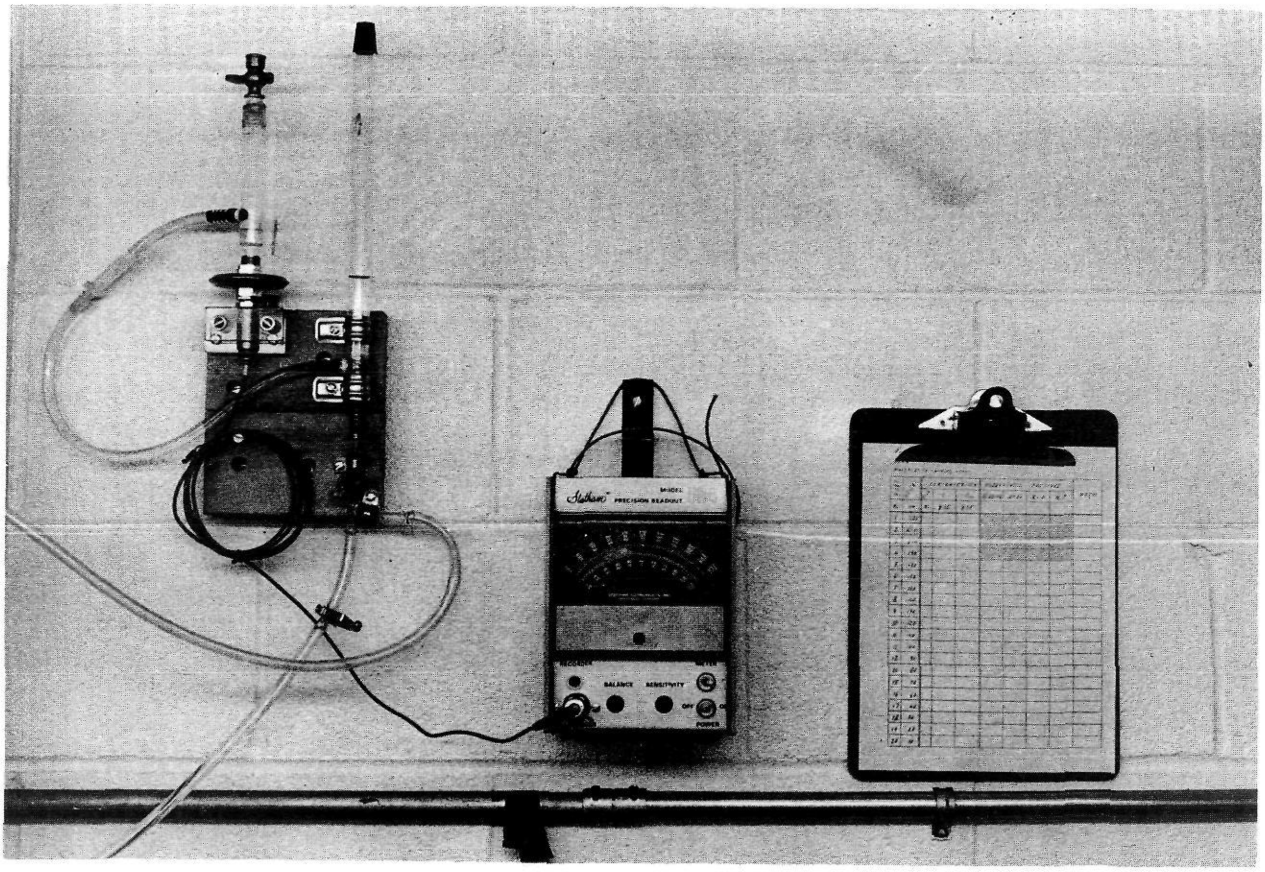


FIGURE 8. SLPA USED BY KOS (3.).

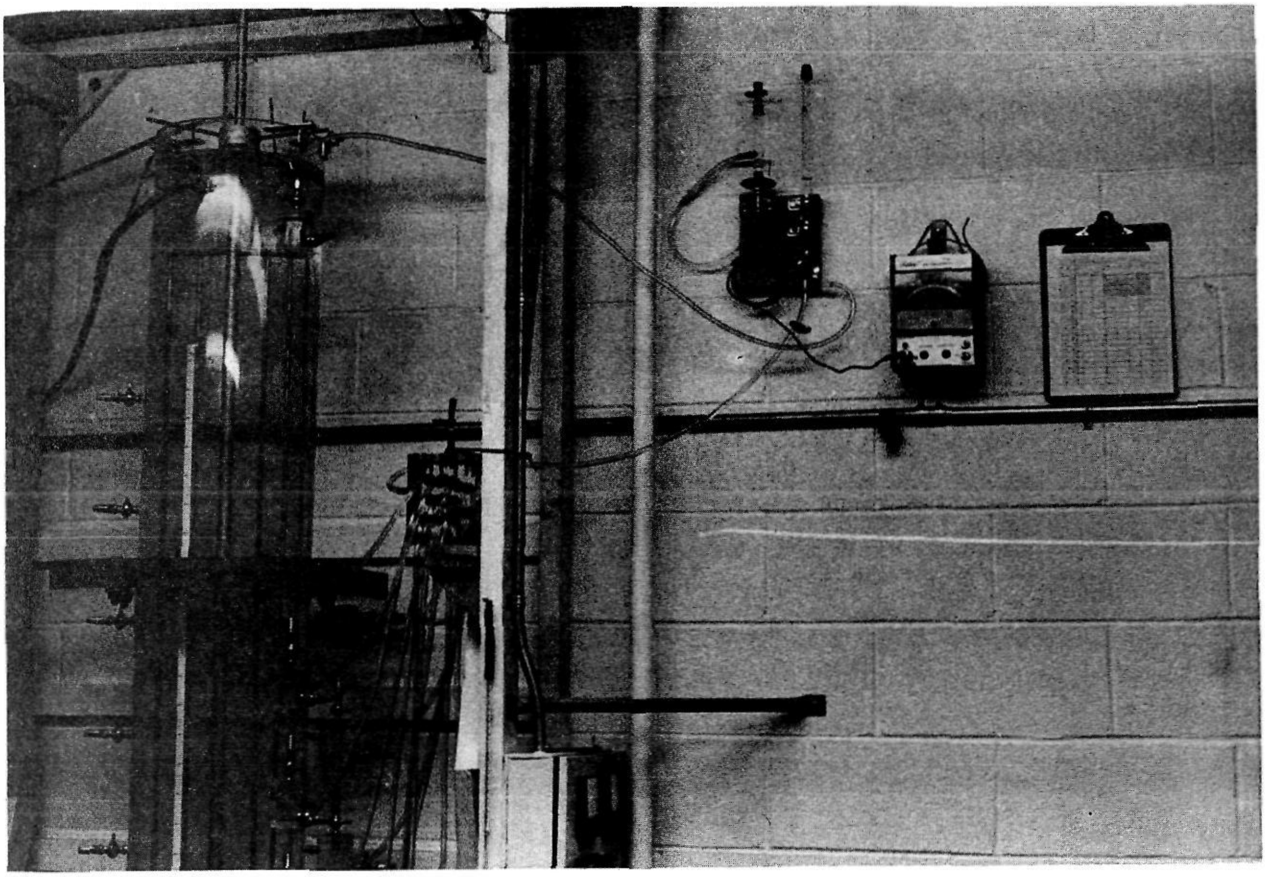


FIGURE 9. THICKENER AND SLPA.

be referred to as $\bar{\sigma}_T$-pressure. This process of determining the liquid level above the datum (by the reference port) is to be carried out before each pressure measurement along the depth of the column. Thus possible error in pressure measurement due to fluctuations in the level of liquid in the thickener is eliminated.

It can be seen from the photographs (Figures 8 and 9) that only one tube from the pressure ports along the thickener is connected directly to the SLPA. This tube is in turn connected to tubing from the individual ports by means of a plexiglass terminal box (see Figure 9). Only the pressure port under consideration is open to the terminal box (by having value B of its separation cell open). All other separation cells must have valve B closed.

We complete our explanation of the pressure measurement apparatus by outlining the function of the glass tube shown to the right of the transducer in the photographs (Figures 8 and 9).

This tube is used to calibrate the readout portion of the SLPA. With the three-way valve closed and the water in this tube initially at the same level as the transducer, small changes are made in the height of the liquid and the corresponding readout responses are recorded. Using this procedure the relationship between pressure (due to the height of liquid above the datum) and readout response is determined. After calibration, this tube, as well as the one directly over the transducer, is filled with water and sealed (one by a valve the other by a rubber stopper). Thus the only liquid motion within the pressure measurement tubing is due to the slight downward deflection of the transducer membrane.

OPERATION OF THE LABORATORY CONTINUOUS THICKENER

It is recommended that the thickener be operating at steady-state during all experimental measurements. Steady state operation by its elimination of a time dependency simplifies the task of data gathering. Before reaching steady state the thickener must go through a transient or "start-up" period. The aim of operation during this period is to attain steady state as quickly as possible. The following is an outline of the procedure to be followed during start-up.

(a) Select a reasonable value for solids loading, G_s. Table 2 gives such values for a number of sludges.

(b) Based on the selected value of solids loading, G_s, and the concentration of the sludge to be thickened, c_{in}, the rate to which the feed pump is to be set, Q_{in}, can be calculated as

$$Q_{in}=G_s/c_{in}$$

(c) Estimate the underflow concentration, c_u, at steady state. As a first estimate select $c_u = 2\ c_{in}$.

(d) Assume the concentration of suspended solids in the overflow, c_o, is negligible.

(e) Calculate the rate to which the underflow pump is to be set, Q_u, based on the selected solids loading, G_s, the estimated underflow concentration, c_u, and the assumption of negligible solids washout.

$$Q_u = G_s/c_u$$

TABLE 2. Design Parameters for Thickening Tanks According to Konicek and Kos [1.]

	Suspended Solids Loading $lbs/day/ft^2$	Thickened Sludge Concentration %	Depth of the Thickening Zone ft
Activated Sludge	1-3*	2-5*	3-5
Primary sludges and mixtures or primary and activated sludges**	1-11	6-13	5-8
Sludges from chemical treatment	∿3	1-3	∿8

*The values are different for various kinds of activated sludge processes.

**Solids ratio of primary to activated sludge 2:1.

(f) The laboratory thickener is then filled with supernatant* from the sludge under study and the inflow and underflow pumps are set to the calculated values of Q_{in} and Q_u. If the estimate of c_u was correct then steady state will be reached within a few days. However if after a few hours a clear interface between the sedimentation and thickening zones does not become established and rise, then the rate of underflow Q_u should be decreased. If on the other hand the interface continues to rise such that the depth of the thickening zone approaches 4 or 5 feet the underflow-rate should be increased. The magnitude of underflow-rate adjustments will be determined by experience. When the interface between the sedimentation and thickening zones remains stationary with respect to column depth and the concentration profile is stable the thickener is operating at steady state.

---

*To decrease start-up time the column can be filled halfway with thickened sludge, the remaining half being filled with supernatant.

MEASUREMENT PROCEDURE

Corresponding to each set of steady state operating parameters are unique pressure and concentration distributions within the thickener. By determining these distributions for several different steady states the consolidation and filtration behavior of a sludge can be measured. Procedure for measuring the pressure and concentration distributions that correspond to a given steady state are outlined below.

Pressure measurement

(1) Obtain a reference pressure reading.

  a) Open valve B of the reference pressure port.

  b) Adjust the three-way valve so that the reference port is open to the SLPA. Observe the meter readout. If the readout is in the range 1 to 2% of total possible deflection - go on to step 2. If the readout does not fall within this range, raise or lower the level of the thickener's adjustable overflow trough until the readout is in the desired range. Record the readout value for the reference port.

(2) Determine liquid phase and total pressure at each port. Pressure measurements are to be made in order of vertical distance from the reference pressure port with the port closest to the reference port being measured first.

a) Slowly open valve B of the pressure port under consideration.

b) If the pressure port is above the interface of the sedimentation and thickening zones go on to step 2c. If the port is within the thickening zone then a sludge membrane or "plug" must be formed in the fitting connecting the separation cell to the thickener (see Figure 7) before liquid phase pressure can be measured. As discussed earlier this is done by carefully opening valve A of the separation cell and allowing liquid to flow out, dropwise until a small amount (1 or 2 ml) of sludge is drawn out of the connector. Great care must be taken in opening both valves A and B of the separation cell. If either is opened in a hasty fashion the fragile solids matrix of the thickening zone will be destroyed and the value measured will be total rather than liquid phase pressure at that point.

c) Measure liquid phase pressure by adjusting the three-way valve so that the tubing connecting the pressure port under consideration and the SLPA is open. Record the value of the SLPA readout. Turn the three-way valve to the reference port. Record the reference value. Again, turn the three-way valve to the pressure port under consideration and record the SLPA readout. Continue this comparison with the reference port until a consistent pressure difference is obtained between the two ports. The pressure difference obtained in this fashion for each port is the liquid phase pressure less the hydrostatic pressure or, to be consistent with Kos' notation, the $\bar{p}$-pressure.

d) Measurement of total pressure for pressure ports within the thickening zone. After determining the $\bar{p}$-pressure at a given pressure port, the total pressure at that port can be measured. First, the structure of the solids matrix in the vicinity of the port must be destroyed by rapidly squeezing the tubing that connects valve B of the port to the terminal box. The total pressure minus hydrostatic pressure (or the $\bar{\sigma}_T$-pressure) is then measured using the same procedure described above for $\bar{p}$-pressure measurement (see 2c). After measuring the liquid phase and total pressures at a given port continue on to the next port and again follow the procedures outlined in steps 2a and 2d. Repeat until measurements have been made at each pressure port.

Plot both $\bar{\sigma}_T$-pressure and $\bar{p}$-pressure against thickener depth as in Figure 10.

SUSPENDED SOLIDS MEASUREMENT

As with pressure measurements, concentration sampling is to begin at the uppermost concentration sampling port and proceed downward. From each port a volume of sample is withdrawn and analyzed for suspended solids.

Note: if the SLPA readout is the same for both liquid phase and total pressure at a particular pressure port then the liquid phase pressure value is in error. There are two possible reasons for such error; one-the sludge membrane or "plug" was not formed properly or two-the solids matrix in the vicinity of the port was inadvertently destroyed, probably by opening or closing a valve too quickly. To obtain a true liquid phase pressure measurement the solids matrix must be allowed to reform (i.e. wait an hour or so) and the measurement repeated.

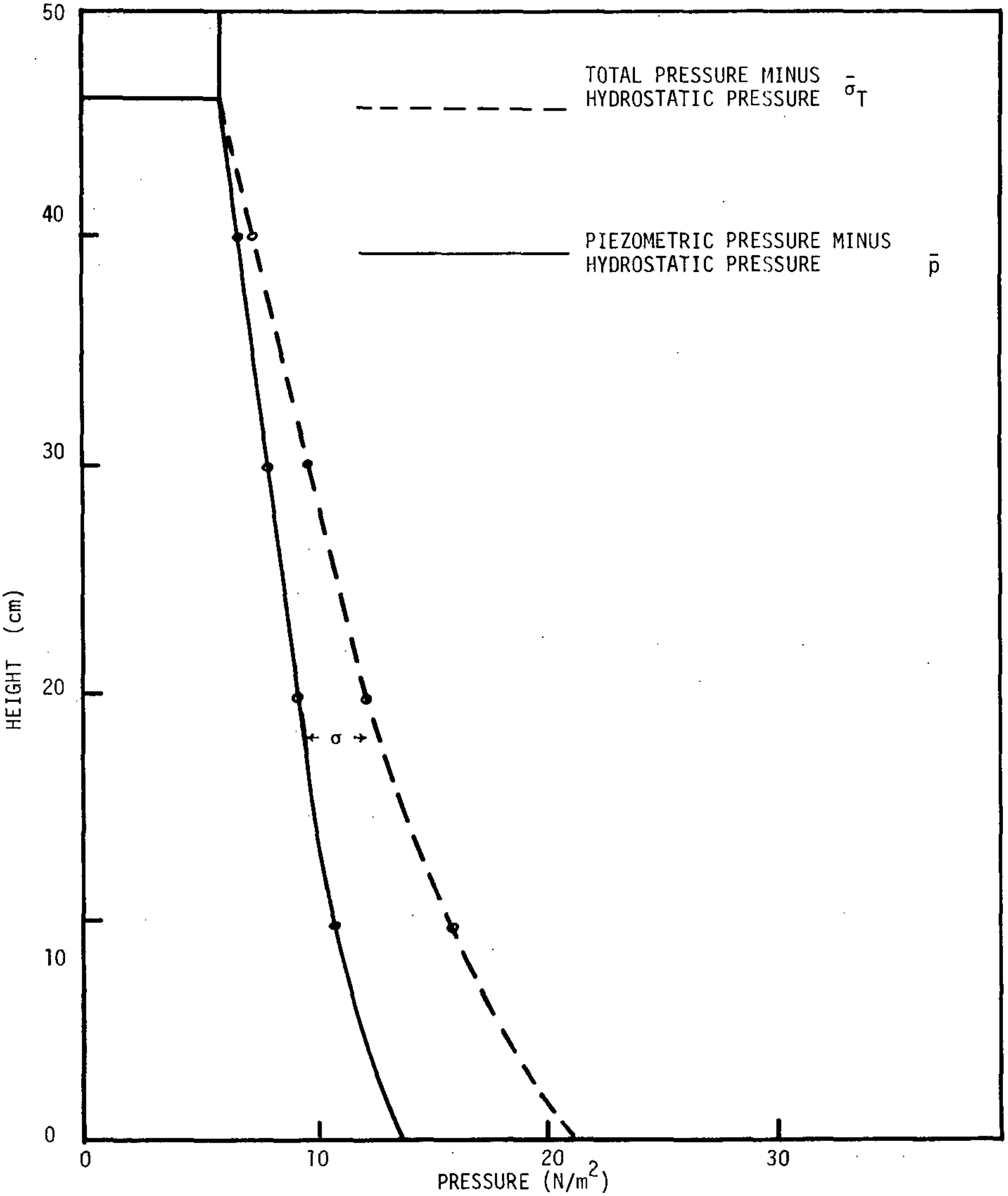


FIGURE 10. AN EXAMPLE OF MEASURED PRESSURE PROFILES IN THE LABORATORY CONTINUOUS GRAVITY THICKENER

The suspended solids concentration profile is obtained by plotting suspended solids concentration against thickener depth as in Figure 11. The interfacial concentration, c_b, is determined by extrapolation.

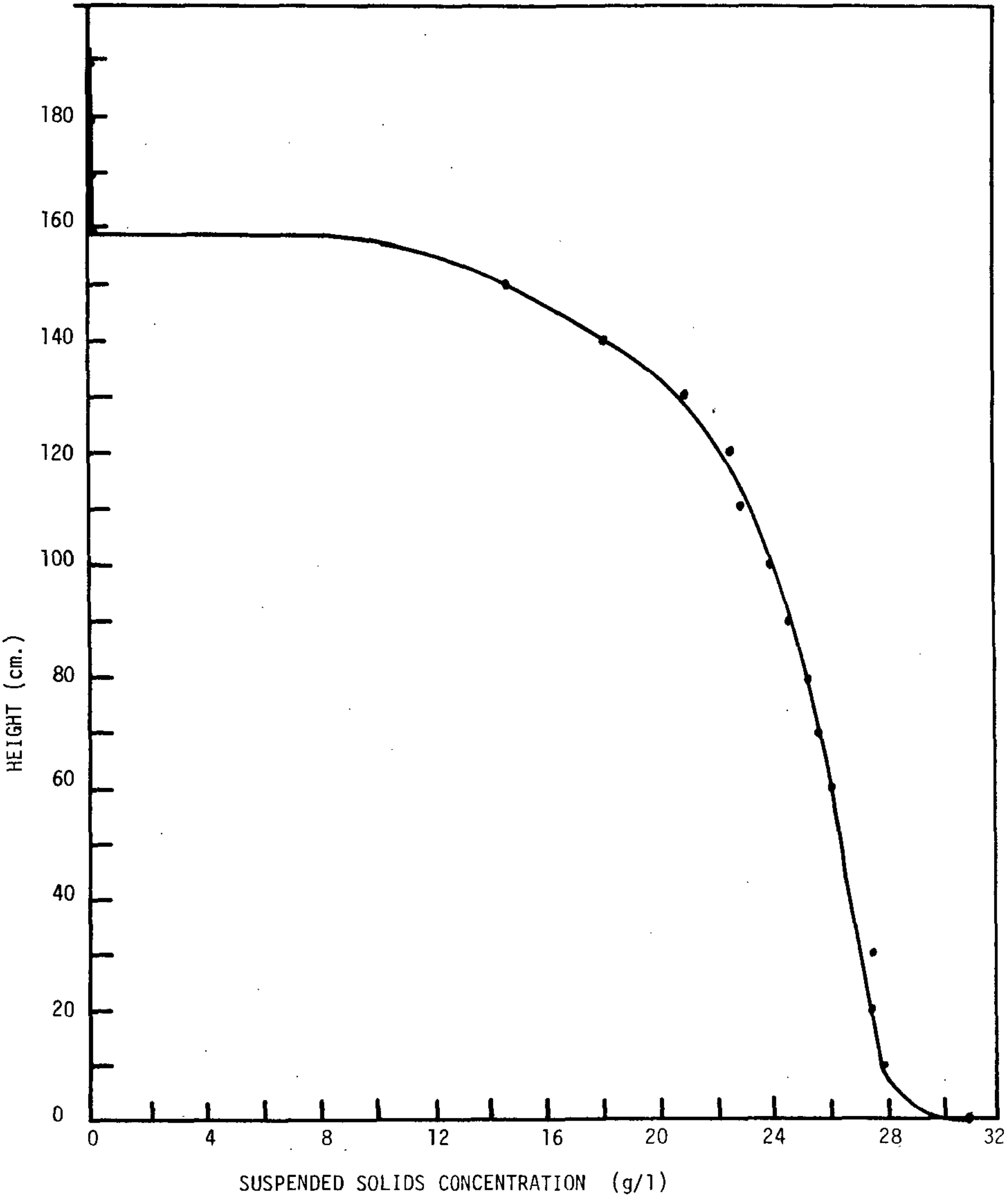


FIGURE 11. AN EXAMPLE OF A MEASURED CONCENTRATION DISTRIBUTION IN THE LABORATORY CONTINUOUS GRAVITY THICKENER

SECTION 4.

ANALYSIS OF DATA

SECTION IV-ANALYSIS OF DATA

DETERMINATION OF THICKENING CHARACTERISTICS

Following the above procedure pressure and concentration profiles that correspond to a particular set of thickener operating conditions can be obtained. To determine the intrinsic conductivity and modulus of linear compressibility of a sludge, pressure and concentration profiles must be obtained experimentally for several sets (at least 5) of thickener operating conditions. Operating conditions can be varied by altering inflow rate or concentration or by changing the underflow rate.

After having pressure and concentrations profiles that correspond to several sets of thickener operating conditions the modulus of linear compressibility and the intrinsic conductivity of the sludges are determined as follows:

MODULUS OF LINEAR COMPRESSIBILITY

For each set of operating conditions match the pressure and concentration profiles as in Figure 12. The effective pressure, σ, at any height within the thickener can be obtained by subtracting the liquid phase from the total pressure at that height. The suspended solids concentration that corresponds to this effective pressure is found from the concentration profile (at the appropriate height). This matching procedure is carried out for each height that corresponds to the height

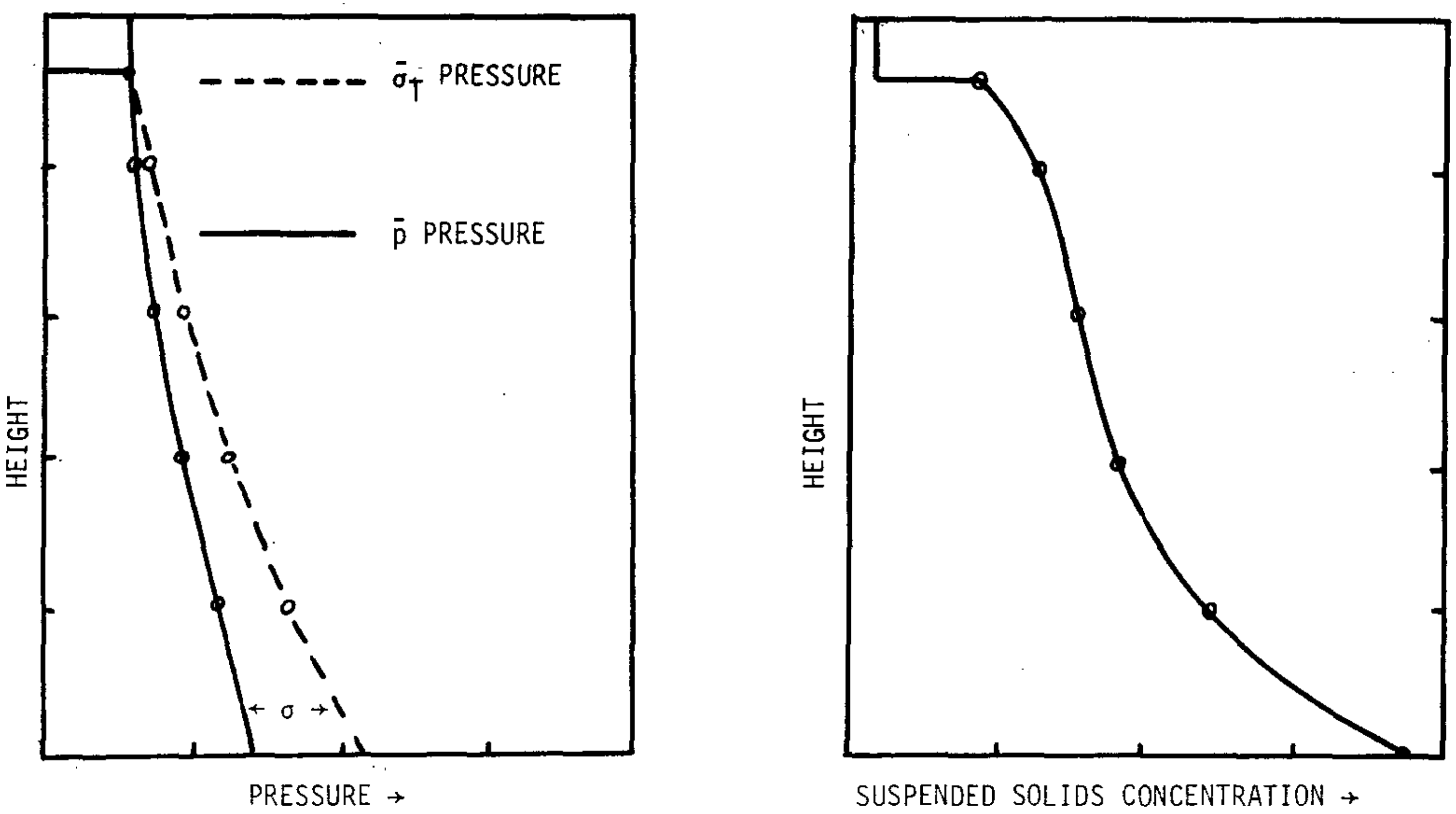


FIGURE 12. PRESSURE AND CONCENTRATION PROFILES CORRESPONDING TO ONE STEADY-STATE RUN OF THE LABORATORY CONTINUOUS THICKENER

of a pressure sampling port within the thickening zone.

Combining the suspended solids concentration versus effective pressure data points for all sets of experimental operating conditions we can obtain a plot such as Figure 13.

A mathematical form is then selected to describe the c-σ relationship (i.e. describe the compressibility of the sludge). Kos (3) chose an equation of the form

$$c = c_b + \overline{A}\sigma^{\overline{B}}$$

to describe mathematically the observed c-σ relationship for his test sludge.

INTRINSIC CONDUCTIVITY

The intrinsic conductivity (or filtration characteristic) of a sludge is a measure of the resistance to flow of escaping liquid offered by the sludge during thickening. The filtration characteristic is a function of both superficial velocity and concentration.

Two relationships are available to express the filtration properties of a sludge. The first relates superficial velocity to liquid pressure drop through the sludge. The second relates the diameter of an imaginary capillary to shear stress on the capillary wall. Although both will be described here the second relationship is more useful for two reasons. First, capillary diameter has a nearly linear dependence on shear stress.

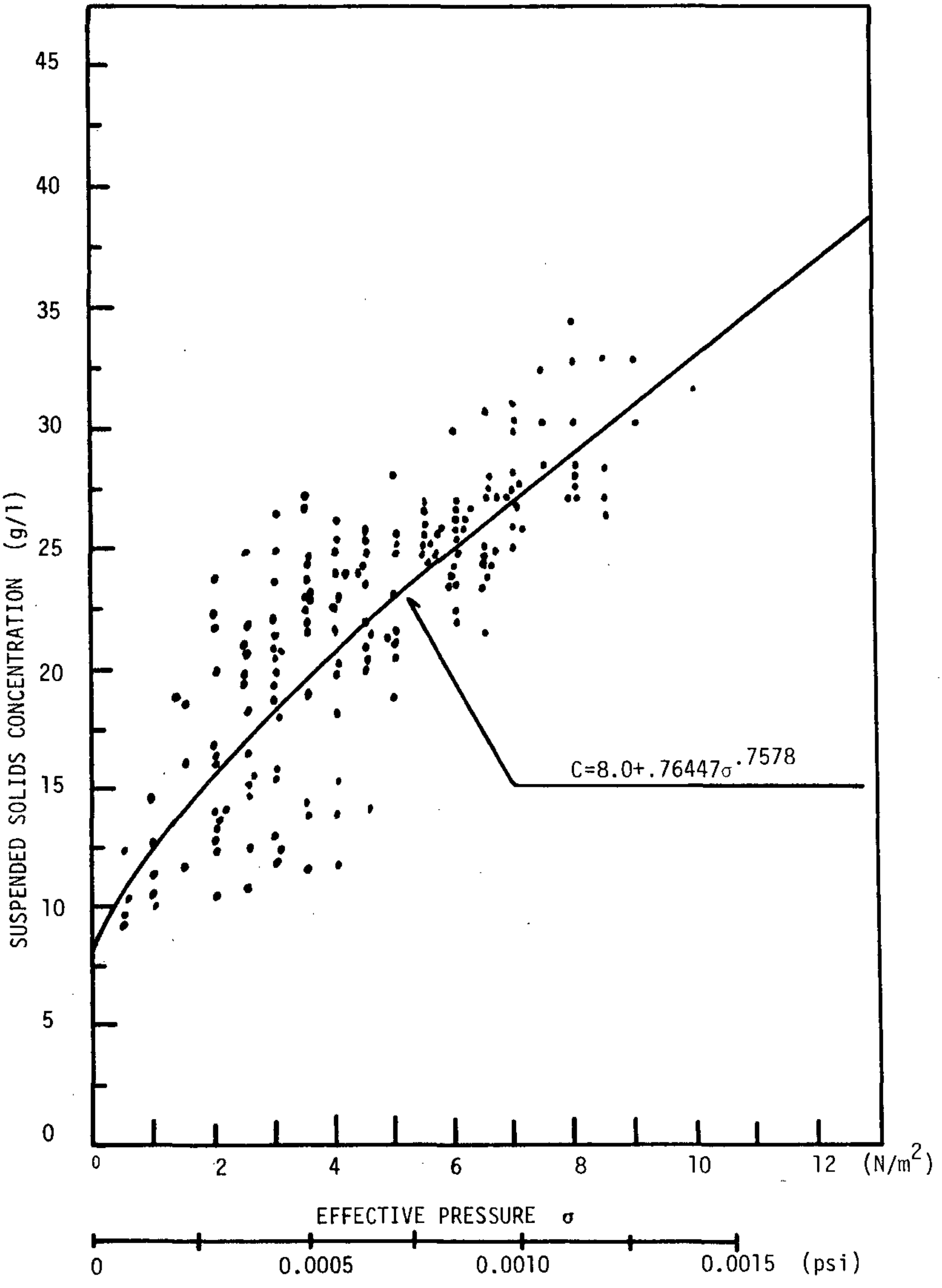


FIGURE 13. SUSPENDED SOLIDS CONCENTRATION AS A FUNCTION OF EFFECTIVE PRESSURE

Thus fewer data points are necessary to get an accurate picture of a sludge's filtration characteristics. Second, the presence of the viscosity term in the expression for shear stress makes it possible to evaluate the effect of temperature on the thickening of a sludge.

To evaluate the filtration characteristics by either method it is first necessary to obtain the superficial velocity versus liquid pressure drop relationship for several values of concentration. This is done by the following procedure.

For each set of operating conditions match the pressure and concentration profiles as in Figure 12. Select a concentration and determine the height at which this concentration occurs from the concentration profile. From the matching $\bar{p}$-pressure profile the value of $d\bar{p}/dx$ at this height [and therefore concentration] can be determined graphically.

The corresponding superficial velocity can be calculated using Equation 8.

$$(v_s - v_\ell)n = G_s \left[\frac{1}{c_i} - \frac{1}{c_u}\right] \tag{8}$$

where c_i is the concentration of interest. Each set of operating conditions gives one data point of the $(v_s - v_\ell)n$ vs. $d\bar{p}/dx$ relationship for a given concentration. Having obtained the one data point go on to the pressure and concentration profiles that correspond to the next set of operating conditions and repeat the above procedure for the concentration of interest and obtain a second data point. Continue until a data point has been obtained for each set of operating conditions (see Figure 14).

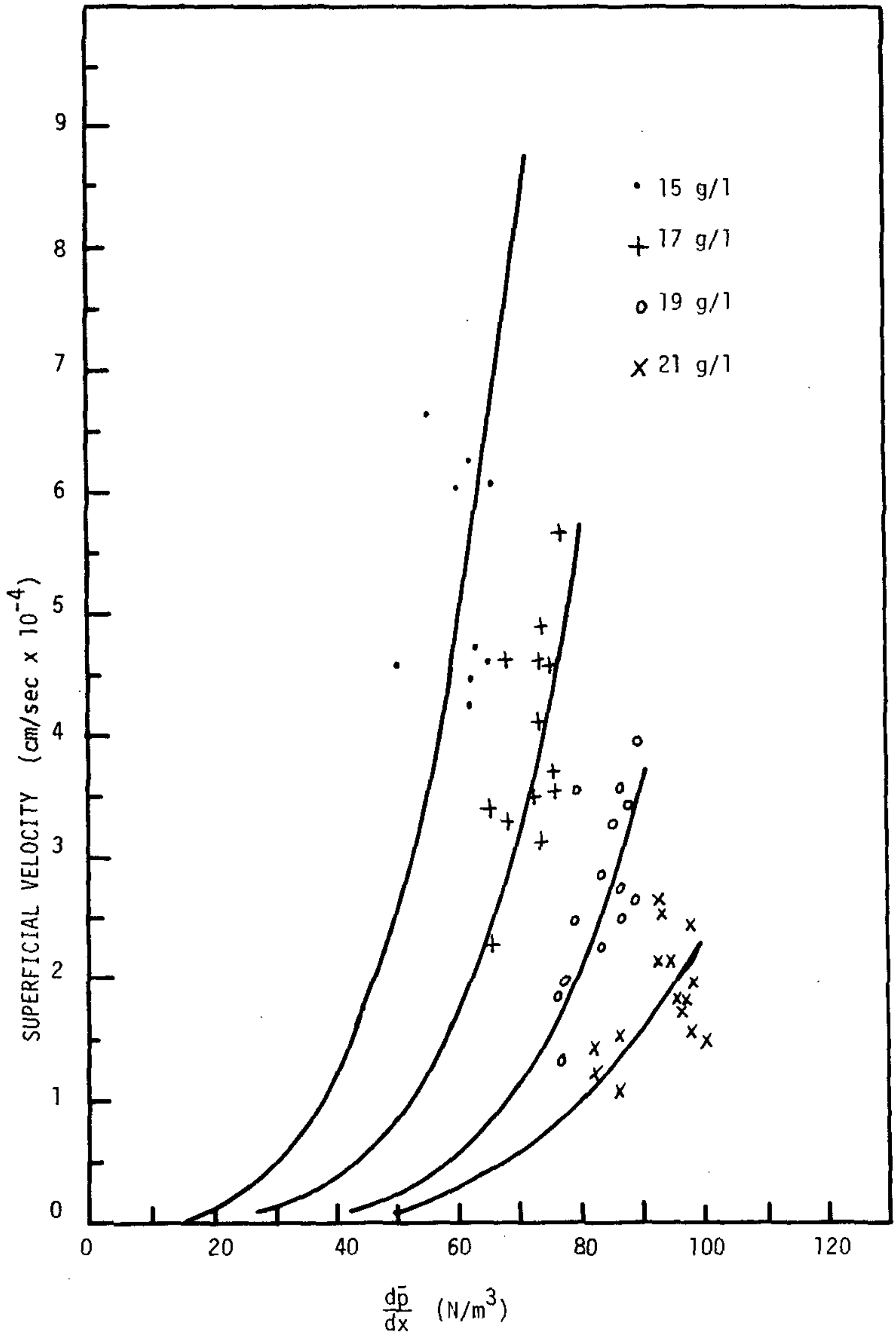


FIGURE 14. PRESSURE GRADIENT, SUPERFICIAL VELOCITY RELATIONSHIP

If the first relationship is to be used to describe the filtration characteristics of the sludge a mathematical form must be selected to describe the $(v_s - v_\ell)n$ vs. $d\bar{p}/dx$ relationship for the concentration of interest. Kos (3) chose the form

$$(v_s - v_\ell)n = A'(d\bar{p}/dx)^{B'} \tag{18}$$

to mathematically describe the experimental data. At incremental concentrations over the range c_b to c_u the constants A' and B' that correspond to a best fit to the data define the filtration characteristics of the sludge. These constants are then used as input data to the computer program of Appendix II.

The second relationship describing filtration behavior is based on an understanding that flow through a deformable porous media (such as sludge) can accurately be modeled as flow through closely-bunched capillary tubes whose diameter varies with flow conditions. The diameter of an imaginary capillary tube, δ, is:

$$\delta = \sqrt{\frac{32k}{n}} \tag{19}$$

where

n = porosity
$k = [\mu(v_s - v_\ell)n]/d\bar{p}/dx$
μ = dynamic viscosity.

The shear stress τ, on the walls of an imaginary tube is given by:

$$\tau = \frac{-8\mu\ (v_s - v_\ell)n}{\delta n} \tag{20}$$

where

τ = shear stress in dynes/cm^2

The diameter of an imaginary capillary tube varies with flow conditions due to the resulting changes in shear stress on the capillary wall. These changes are nearly linear and can be expressed by the equation:

$$\delta = \delta_0 - K\tau \qquad (21)$$

Using the second method the filtration characteristics are defined for a particular sludge when the constants δ_0 and K have been calculated for a range of concentrations.

COMPUTER SIMULATION OF GRAVITY THICKENING

An analysis of transport phenomena within a gravity thickener during steady state continuous operation yielded the equations summarized in Table 1. Before these equations can be used to predict the thickening behavior of a particular sludge, that sludge's filtration and consolidation characteristics must be determined experimentally.

The consolidation characteristic $\bar{a}$ represents the relationship between sludge concentration and effective (or interparticle) pressure. For his test sludge Kos (3) found this relationship to have the form

$$c = c_b + \bar{A}_{\sigma}^{\bar{B}}$$

Thus the consolidation characteristic for this sludge is defined when the constants c_b, $\bar{A}$ and $\bar{B}$ have been determined experimentally.

The filtration characteristic k is a measure of the resistance offered by a particular sludge to the upflow of liquid displaced by the downward movement of sludge solids during consolidation. The filtration characteristic is defined for a particular sludge when the constants A' and B' or δ_0 and K have been experimentally determined for a range of concentrations.

The experimentally determined constants that define the filtration and consolidation properties of a sludge can be used with the transport equations to predict the thickening behavior (and therefore the design parameters) for that sludge.

In order to ease calculation of design parameters two computer programs have been written. The only difference in the programs is in the relationship used to describe the filtration properties of the sludge. One uses A''s and B''s as input constants, the other δ_o's and κ's.

For a given set of operating conditions (i.e. G_s and c_u) each program uses the equations of Table 1 and the experimental data to predict the pressure and concentration profiles that would occur during steady state thickening of the corresponding sludge. A detailed description of the programs is given in Appendix II.

SECTION 5.

PRACTICAL APPLICATIONS

SECTION V - PRACTICAL APPLICATION

Rational design of gravity thickeners (or clarifiers) must be based on an understanding of the interrelationships that exist among such factors as solids loading, underflow concentration and the height of the thickening zone. The mathematical model presented in this report can serve as a basis for such an understanding.

To facilitate use of the model the coupled equations of Table 1 have been solved numerically using a digital computer.* By specifying a solids loading and a desired underflow concentration, the computer calculates the corresponding depth of the thickening zone. Repeated calculations of this type, varying solids loading and underflow concentration, yield the information needed to construct a graph such as Figure 15 of this report. For the test sludge used all possible design options among area (solids loading) depth and underflow concentration are given in Figure 15.

The data points shown in Figure 15 represent measured values obtained by operating the laboratory gravity thickener. The associated numerical values represent suspended solids loadings. It can be seen that the values predicted using the computer are in close agreement with those measured in the laboratory.

To emphasize the practical usefulness of Figure 15 to design and operating engineers, several examples are given.

---

* Details of this solution including example programs are given in Appendix II of this report.

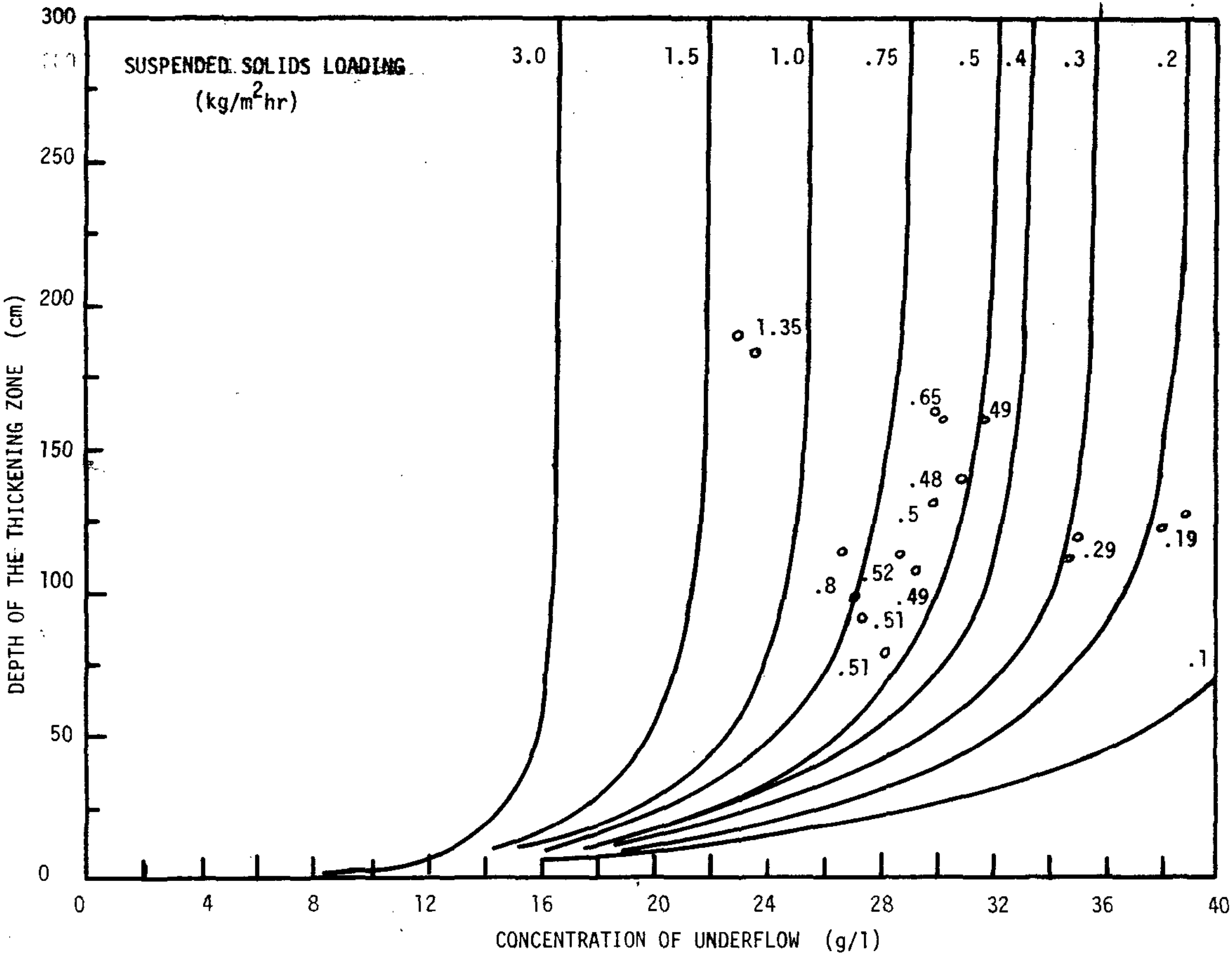


FIGURE 15. DEPTH OF THE THICKENING ZONE VERSUS UNDERFLOW CONCENTRATION AND SOLIDS LOADING

Example A.

An existing plant has a gravity thickener with a diameter of 20 meters. Inflow to the unit is 7.0×10^4 ℓ/hr (0.44 mgd) at a suspended solids concentration of 7 g/ℓ. Can the underflow rate be regulated to achieve and underflow suspended solids concentration of 24 g/1?

SOLUTION:

Determine the suspended solids loading to the unit

$$\text{SS loading} = \frac{(7.0 \times 10^4 \ell/\text{hr})\ (7\ \text{g}/\ell)\ (\text{kg}/1000\ \text{g})}{\pi (20\ \text{m})^2/4}$$

Figure 15 shows that the maximum underflow SS concentration possible for a loading of 1.56 kg/m^{2}hr is approximately 21 g/ℓ. This value represents the underflow concentration at which a curve corresponding to a loading of $1.56 \frac{\text{kg}}{\text{m}^2\text{hr}}$ becomes vertically asymptotic. Therefore the unit under study is not capable of achieving an underflow concentration of 24 g/ℓ at the specified loading.

Example B

A gravity thickener with a loading of 0.75 kg/m^{2}hr is operated such that the underflow concentration is 18 g/ℓ. This level of performance is deemed unacceptable by the plant operator. He therefore undertakes an experimental program to determine the maximum underflow concentration attainable from the thickener under its loading of 0.75 kg/m^2hr.

The iterative experimental procedure consisted of decreasing underflow rate, allowing the system to come to steady state and measuring the new (presumably higher) underflow concentration. Eventually the operator would determine the minimum allowable underflow rate and the resulting maximum attainable underflow concentration for this thickener. Enroute the depth of the thickening zone would, at some time, probably exceed the height of the overflow weir causing drastic deterioration of effluent quality. In addition such an experimental procedure would be quite time consuming with several days often necessary for the establishment of a new steady state.

The mathematical model presented in this report offers an attractive alternative to such experimental work. Once the filtration and consolidation characteristics of a particular sludge have been determined, the computer can generate the information necessary to construct a graph such as Figure 15.

By inspection of Figure 15 we can follow the progress of the experimental procedure described earlier in this example. Initially the thickener is operated so that the underflow concentration is 18 g/ℓ. A decrease in underflow rate causes the depth of the thickening zone (i.e. height of the sludge layer) to increase. This added mass in the thickening zone causes an increase in compressive (or interparticle) stress throughout the thickening zone. As the strata throughout the sludge layer are compressed concentration increases as given by Equation 13. Thus a new steady state underflow concentration [say 22 g/ℓ] corresponding to our loading and new underflow rate is achieved in the thickener.

---

*Throughout this hypothetical experiment the loading remains constant at 0.75 kg/m^2hr.

Again underflow rate is decreased. Depth of the thickening zone now increases to 75 cm and the resulting underflow concentration is approximately 26 g/ℓ.

A further slight decrease in underflow rate results in an underflow concentration of 28 g/ℓ and a thickening zone height of approximately 130 cm. As the overflow weir height of our thickener is about 230cm (approximately 7 feet), no significant washout of solids is evident. Therefore, the operator decreases the underflow rate again in an effort to increase underflow concentration. This was one decrease too many. The height of the sludge layer rises above the overflow weir causing a substantial washout of solids. This dramatic increase in sludge layer height with increases in underflow concentration above 28 g/ℓ is clearly predicted by Figure 15.

The primary reason for this limiting underflow concentration is thought to be the interdependence of superficial velocity and concentration. Increases in superficial velocity which generally correspond to increases in underflow concentration for a given loading, cause the drag force of the liquid phase on the solid phase to increase (Equation 9). As new matter becomes part of the thickening zone its mass is supported primarily by a combination of viscous drag forces and interparticle contacts. It is only the portion of mass supported by interparticle contact that acts to compress the underlying solids matrix. The asymptotic nature of the curves of Figure 15 reflects the fact that at some point the forces that result from increased sludge layer height are balanced by the drag forces that result from increased superficial velocity. Beyond this point further increases in sludge layer height have virtually no effect on underflow concentration.

Example C

Loading to an existing gravity thickener is to be increased from 0.75 kg/m^2hr to 1.0 kg/m^2 hr. To what height will the sludge layer rise if underflow is maintained at its current concentration of 24 g/ℓ?

Solution:

With an underflow concentration of 24 g/ℓ and a loading of 0.75 g/ℓ , Figure 17 indicates a thickening zone depth of about 50 cm. When solids loading is increased to 1.0 kg/m^2 hr and underflow concentration is controlled at 24 g/ℓ, thickening zone depth as predicted by Figure 15 is approximately 85 cm.

Example D

Suspended solids loadings to a thickener vary between 0.75 kg/m^2 hr and 1.5 kg/m^2 hr. If the height of the thickening zone is not to exceed 200 cm what is the range of maximum underflow concentrations attainable from the unit?

Solution:

A horizontal line crossing the ordinate of Figure 15 at the 200 cm mark intersects the 1.5 kg/m^2hr curve at an underflow concentration of 22 g/ℓ; the 0.75 kg/m^2 hr curve is intersected at an underflow concentration of 29 g/ℓ. These concentrations represent the maximum values under the problem statement.

Example E

A design engineer is asked to determine the surface area of a proposed gravity thickener. Previous studies dictate that the maximum thickening zone depth be 150cm. Design inflow is $7.0\times10^4 \ell/\text{hr}$ (0.44 MGD) at a suspended solids concentration of 7 g/ℓ. Desired underflow concentration is 28 g/ℓ.

Solution:

The curve for a suspended solids loading of 0.75 kg/m^2 hr passes through the point: depth = 150 cm, concentration = 28 g/ℓ. Thus we wish to design for this loading

$$\text{SS loading} = \frac{\text{mass of SS into thickener/hr}}{\text{thickener area}}$$

$$\text{thickener area} = \frac{\text{mass of SS in per hour}}{\text{SS loading}}$$

$$\text{thickener area} = \frac{(7.0 \times 10^4 \ell/\text{hr})\ (7\ \text{g}/\ell)\ (\text{kg}/1000\ \text{g})}{0.75\ \text{kg/m}^2\ \text{hr}}$$

$$\text{thickener area} = 653.33\ \text{m}^2$$

thickener diameter = 28.84 m

By examining the previous examples and from experience we know that it is often useful to know the maximum underflow concentration attainable for a given solids loading. Conversely the maximum solids loading that still allows a desired underflow concentration is of interest. Although this information can be extracted from Figure 15 it is of sufficient interest to warrant its own graph.

Data for Figure 16 is obtained by noting the point at which each curve of Figure 15 become vertical. The underflow concentration at which each curve becomes vertical is plotted on Figure 16 against the corresponding loading. Several of the examples can be solved more readily using Figure 16.

Example A: After calculating the SS loading of 1.56 kg/m^2 hr it is apparent from Figure 16 that the maximum underflow concentration is about 21 g/ℓ.

Example B: By inspection of Figure 16 the maximum underflow concentration corresponding to a loading of 0.75 g/ℓ is approximately 28 g/ℓ.

Finally, the dashed line of Figure 16 represents information obtained for the same sludge using conventional methods of design (i.e. batch thickening tests and the Yoshioka graphical method of analysis). A design engineer interested in achieving an underflow concentration of 20 g/ℓ using conventional design methods would choose a loading rate of 3.8 kg/m^2 hr. Analysis by the new theory shows that for a loading rate of 3.8 kg/m^2 hr the resultant underflow concentration would be only 15 g/ℓ. Figure 16 shows clearly that for this sludge, conventional methods can lead to significant design error.

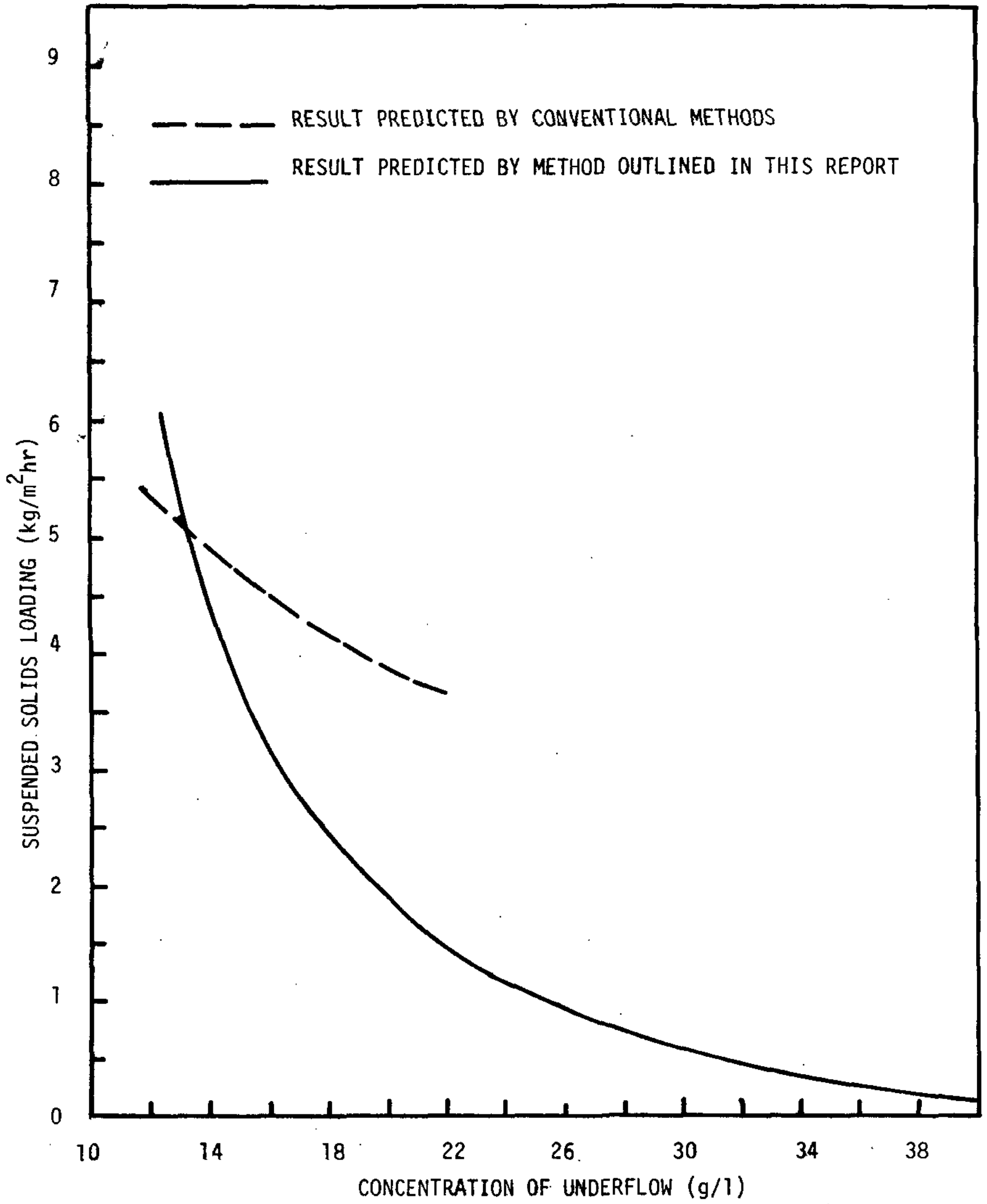


FIGURE 16. MAXIMUM SUSPENDED SOLIDS LOADING AS A FUNCTION OF UNDERFLOW CONCENTRATION

APPENDIX 1.

LIST OF SYMBOLS

APPENDIX I - LIST OF SYMBOLS

The following symbols are used in this report:

Symbol		Dimensions
A	Area	L^2
$\bar{A}$	Constant from Equation 17	$L^{-2}T^2$
A'	Constant from Equation 18	$M^{-1}L^2T^2$
a	Modulus of linear compressibility ($a=d\rho_s/d\sigma$)	$L^{-2}T^2$
$\bar{a}$	Modulus of linear compressibility ($\bar{a} = dc/d\sigma$)	$L^{-2}T^2$
$\bar{B}$	Constant from Equation 17	---
B'	Constant from Equation 18	---
c	Concentration of suspended solids	ML^{-3}
c_b	Concentration of sludge at the boundary between zone settling and channeling	ML^{-3}
c_u	Concentration of underflow	ML^{-3}
C_{in}	Concentration of inflow	ML^{-3}
C_o	Concentration of overflow	ML^{-3}
d_s	Density of solid particles	ML^{-3}
d_l	Density of liquid	ML^{-3}
d_{sl}	Density of sludge	ML^{-3}
F	Resistivity	$ML^{-3}T^{-1}$

Symbol		Dimensions
L	Suspended Solids Loading	$ML^{-2}T^{-1}$
G_s	Mass flux of the solid phase	$ML^{-2}T^{-1}$
G_l	Mass flux of the liquid phase	$ML^{-2}T^{-1}$
g_x	Acceleration of gravity in the x direction	LT^{-2}
k	Intrinsic conductivity	L^2
n	Porosity ($n = \rho_\ell/d_\ell$	---
p	liquid phase pressure	$ML^{-1}T^{-2}$
$\overline{p}$	Piezometric pressure minus the hydrostatic pressure	$ML^{-1}T^{-2}$
Q	Flow rate	L^3T^{-1}
Q_{in}	Inflow	L^3T^{-1}
Q_u	Underflow	L^3T^{-1}
Q_o	Overflow	L^3T^{-1}
t	Time	T
v_s	Velocity of solids matrix at point x	LT^{-1}
v_ℓ	Velocity of liquid at point x	LT^{-1}
$(v_s-v_\ell)\ n$	Superficial velocity	LT^{-1}
x	The spatial or Eulerian coordinates	L
δ_o	Diameter of the capillary at zero shear stress	L

Symbol		Dimensions
δ	Diameter of the capillary	L
ρ_s	Bulk mass density of solids	ML^{-3}
ρ_ℓ	Bulk mass density of liquid	ML^{-3}
μ	Dynamic viscosity	$ML^{-1}T^{-1}$
σ	Effective (interparticle pressure)	$ML^{-1}T^{-2}$
σ_T	Total pressure	$ML^{-1}T^{-2}$
$\overline{\sigma}_T$	Total pressure minus the hydrostatic pressure	$ML^{-1}T^{-2}$
τ	Shear stress	$ML^{-1}T^{-2}$

APPENDIX 2.

COMPUTER PROGRAMS AND DESCRIPTIONS

APPENDIX II - COMPUTER PROGRAMS

Both programs calculate the pressure and concentration profiles in the thickening zone of a continuously operated thickener under a given set of operating conditions. Execution begins at the top of the thickening zone and proceeds in Δx increments until the required underflow concentration c_u is reached. If the chosen operating conditions (G_s and c_u) are unrealistic or yield a very high thickening zone the program will do calculations only for first 500 depth increments.

The programs are actually two modifications of a single program. The first modification uses equation 18 for expressing the filtration characteristics of sludge, while the second modification uses the equation 21.

The computer program flow diagram is shown in Figure 17 and the listing of both modifications together with the computer variable listing is at the end of the Appendix. The parameters in this listing are for the alum sludge measured by Kos (3). In order to use the program for any other suspension, the parameters of that suspension must be entered into the program. Following is the list of necessary values:

> The interfacial concentration c_b (statement 60), density of solids d_s (70), density of liquid d_ℓ(75), the consolidation characteristics $\bar{A}$ and $\bar{B}$ (statements 95 and 100), dynamic viscosity of liquid (80). When equation 18 is used for expressing the filtration properties, modification one and the constant A' and B' of equation 18 for various concentrations should be used.
>
> When the filtration properties are expressed by equation 21,

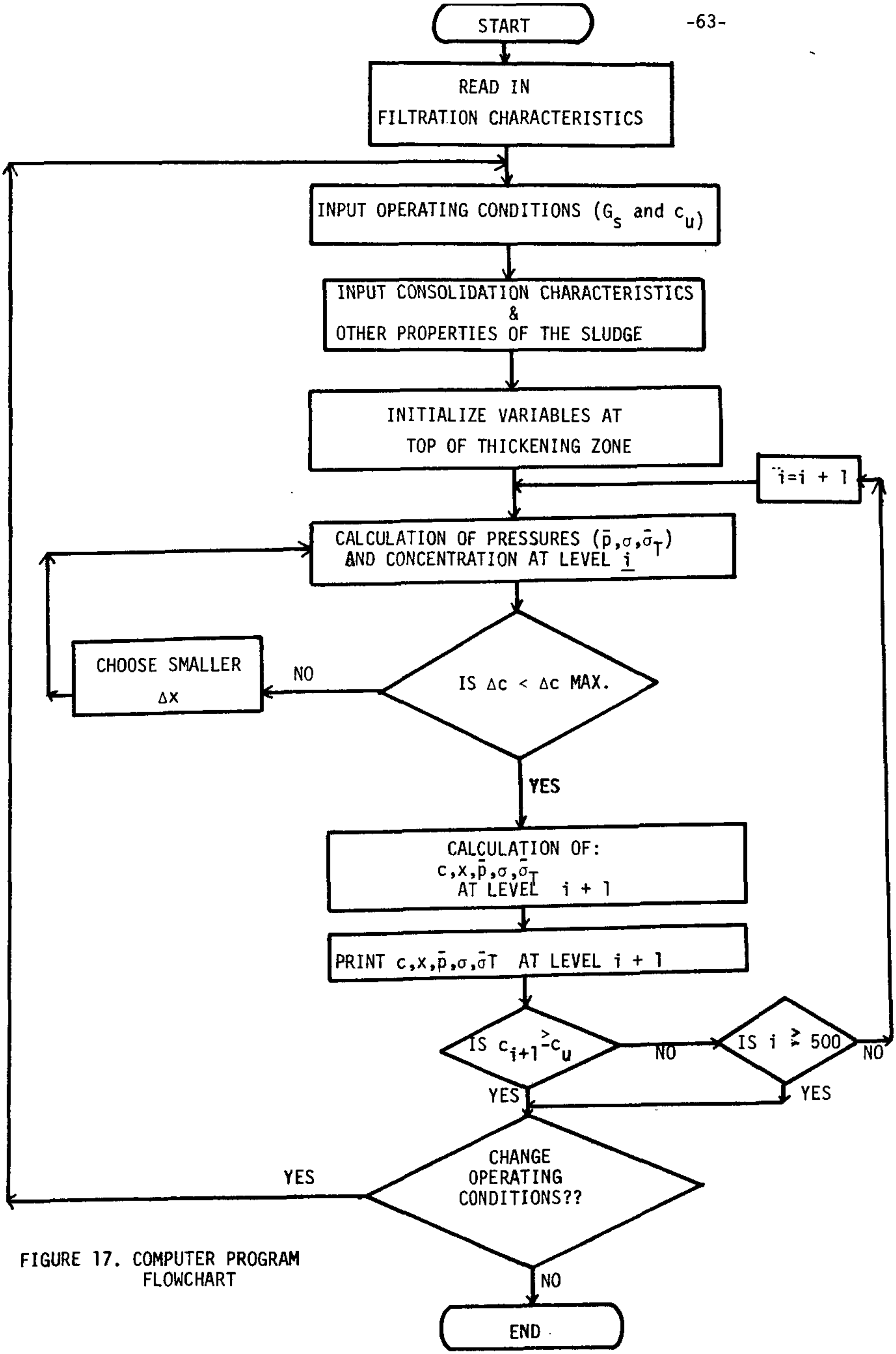


FIGURE 17. COMPUTER PROGRAM FLOWCHART

then modification two and the constants δ_o and K should be used. The values of constants A' and B' or δ_o and K for a given concentration c_i are entered at the end of the program under statement number 800 + i (for example, values for concentration 15 g/ℓ will be entered under the statement number 815.)

The number of these statements should also be changed in the DO-loop statement 27. Should a different relationship than that of equations 18 or 21 be used the entire subroutine (statements 600-635) must be changed.

The program uses the c-g-s system of units.

Computer Variable Listing and Definition

Variable Name	Definition
A	Constant from equation 17($\bar{A}$), cm sec^2/g
AJSL	Superficial velocity $(v_s - v_\ell)n$, cm/sec
AM	Dynamic viscosity of liquid (at the temperature during experiments), g/cm sec
AMT	Dynamic viscosity of liquid (at the simulated conditions), g/cm sec
B	Exponent from equation 17($\bar{B}$), dimensionless
C	Suspended solids concentration at depth i, g/ℓ
CN	Suspended solids concentration at depth i + 1, g/ℓ
CO	Interfacial concentration c_b, g/ℓ
CU	Concentration of underflow c_u, g/ℓ
DL	Density of liquid phase, g/ℓ
DS	Density of solid phase, g/ℓ
G	Acceleration of gravity, cm/sec^2
GS	Suspended solids loading of the thickener, kg/m^2/hr
P	$\bar{p}$ - pressure; piezometric pressure minus the hydrostatic pressure, g/cm sec^2
SIG	Effective pressure (σ), g/cm sec^2

Variable Name	Definition
SIGT	σ_T-pressure; total pressure minus the hydrostatic pressure, g/cm sec^2
T	Value of constant A' from 18 (Program 1); value of constant δ_0 from equation 21 (Program 2).
TT	Exponent from equation 18 (B'), (Program 1); value of constant K from equation 21 (Program 2).
X	Depth of the thickening zone, cm
XC	Increment of concentration (Δc), g/ℓ
XP	Increment of $\bar{p}$-pressure ($\Delta\bar{p}$), g/cm sec^2
XPX	$\Delta\bar{p}/\Delta x$, g/cm^2 sec^2
XSIG	Increment of effective pressure ($\Delta\sigma$), g/cm sec^2
XSIGT	Increment of $\bar{\sigma}_T$-pressure ($\Delta\bar{\sigma}_T$), g/cm sec^2
XX	Increment of depth (Δx), cm
XXO	Maximum increment of depth (Δx), cm

COMPUTER PROGRAM: MODIFICATION 1.

```
10  PROGRAM DEPTH
15* PROGRAM CALCULATES THE CONCENTRATION, THE PIEZOMETRIC PRESSURE
16* THE EFFECTIVE PRESSURE AND THE TOTAL PRESSURE DISTRIBUTIONS
17* IN THE CONTINUOUS GRAVITY THICKENER.
18* FOR ANY SLUDGE PUT APPROPRIATE VALUES FOR CO (STATEMENT 60),
19* DS (70), DL (75), A (95), B (100), AND CHANGE THE XPX-AJSL RELATIONSHIP
20* IN THE SUBROUTINE (610-630, AND 800-850) + (27,28).
21* IF THE TEMPERATURE IS DIFFERENT FROM THAT DURING EXPERIMENT CORRECT
22* AMT(85) AND AM (80).
25  DIMENSION T (50), TT (50)
27  DO 28 J=1,33
28  READ, T (J+7),TT (J+7)
30  PRINT 35
35  FORMAT (*TYPE GS(KG/M**2/HR), CU (G/L*)
40  INPUT, GS, CU
60  CO=8.0
65  G=981.
70  DS=1.921506
75  DL=.998425
80  AM=.010
85  AMT=.010
90  XXO=1.
95  A=.76446986
100 B=.75781267
150 X=0.0
155 P=0.0
160 SIG=0.0
165 SIGT=0.0
170 C=CO
175 AA=G*((DS-DL)/DS)
200 PRINT 205
205 FORMAT (//6X,*X(CM)*,3X,*C(G/L)*,2X,*P(DYN/CM**2)*,2X*SIG(DYN/CM**2)
206C*,2X,*SIGT(DYN/CM**2)*)
210 PRINT 220,X,C,P,SIG,SIGT
220 FORMAT (2X,2F9.2,3F14.2)
300 DO 500 I=1,500
310 XX=XXO
320 AJSL=(GS/36.)*((1./C)-(1./CU))
330 CALL XPSUBR (C,AJSL,XPX,T,TT)
350 XP=XPX*XX*(AMT/AM)
360 XSIGT = AA*C*.001*XX
370 XSIG =XSIGT -XP
380 BB=SIG+XSIG
390 CN=8.0+A*(BB**B)
400 XC=CN-C
410 IF(XC.LE.1.0)GO TO 445
415 K=IFIX (XC)+1
420 XX=XX/FLOAT (K)
425 TO TO 350
445 C=CN
450 X=X+XX
```

```
455 P=P+XP
460 SIG=SIG+XSIG
465 SIGT =SIGT+XSIGT
470 PRINT 220,X,C,P,SIG,SIGT
495 1F(C.GE.CU)GO TO 510
500 CONTINUE
510 PRINT 515
515 FORMAT (*DO YOU WISH TO CHANGE CONDITIONS?
516CTYPE 1-YES, 0-NO*)
520 INPUT, J
525 IF(J.EQ.1) GO TO 30
540 END
600 SUBROUTINE XPSUBR (C,AJSL,XPX, T,TT)
605 DIMENSION T (50),TT (50)
610 L=IFIX(C)
615 M=L+1
620 CXPL=(AJSL*.1*T(L))**TT(L)
625 CXPM =(AJSL*.1*T(M))**TT(M)
630 XPX=CXPL+ (CXPM-CXPL)* (C-FLOAT (L))
640 RETURN
650 END
700 ENDPROG
808 10.0E5   .1977866358
809 10.0E5   .2302253632
810 10.0E5   .2637260433
811 10.0E5   .2982118366
812 10.0E5   .3336180709
813 10.0E5   .3698894427
814 10.0E6   .2714462644
815 10.0R7   .2129242363
816 10.0E7   .2265930477
817 10.0E7   .2402311732
818 10.0E7   .2538404827
819 10.0E8   .2089698176
820 10.0E9   .1753651912
821 10.0E10  .1545507829
822 10.0E11  .1375567432
823 10.0E12  .1245261325
824 10.0E12  .1283270685
825 10.0E12  .1320817837
826 10.0E12  .1357926570
827 10.0E12  .1394618582
828 10.0E12  .1430913741
829 10.0E12  .1466830292
830 10.0E12  .1502385043
831 10.0E12  .1537593526
832 10.0E12  .1572470125
833 10.0E12  .1607028201
834 10.0E12  .1641280185
835 10.0E12  .1675237678
836 10.0E12  .1708911520
837 10.0E12  .1742311865
838 10.0E12  .1775448239
839 10.0E12  .1808329595
840 10.0E12  .1840964362
```

COMPUTER PROGRAM: MODIFICATION 2.

```
10 PROGRAM DEPTH
15*PROGRAM CALCULATES THE CONCENTRATION, THE PIEZOMETRIC PRESSURE,
16*THE EFFECTIVE PRESSURE AND THE TOTAL PRESSURE DISTRIBUTIONS
17*IN THE CONTINUOUS GRAVITY THICKENER.
18*FOR ANY SLUDGE PUT APPROPRIATE VALUES FOR CO (STATEMENT 60),
19*DS(70),DL(75),A(95),B(100), AND CHANGE THE XPX-AJSL RELATIONSHIP
20*IN THE SUBROUTINE (610-630, AND 800-850)+(27,28).
21*IF THE TEMPERATURE IS DIFFERENT FROM THAT DURING EXPERIMENT CORRECT
22*AMT(85) AND AM (80).
25 DIMENSION T(50), TT(50)
27 DO 28 J=1,33
28 READ, T (J+7),TT (J+7)
29 PRINT, T (8)
30 PRINT 35
35 FORMAT (*TYPE GS (KG/M**2/HR),CU(G/L*)
40 INPUT, GS,CU
60 CO=8.0
65 G=981.
70 DS=1.921506
75 DL=.998425
80 AM=.010216
90 XXO=1.
95 A=.76446986
100 B = .75781267
150 X = 0.0
155 P=0.0
160 SIG=0.0
165 SIGT=0.0
170 C=CO
175 AA G*((DS-DL)/DS)
200 PRINT 205
205 FORMAT (//6X,*X(CM)*,3X,*C(G/L)*2X,*P(DYN/CM**2)*,2X*SIG (DYN/CM**2)
206C*,2X,*SIGT (DYN/CM**2)*)
210 PRINT 220,X,C,P,SIG,SIGT
220 FORMAT (2X,2F9 2,3F14.2)
300 DO 500 1=1,500
310 XX XXO
320 AJSL=(GS/36.)*((1./C)-(1./CU))
330 CALL XPSUBR (C,AJSL, XPX, T,TT, AM)
350 XP=XPX*XX
360 XSIGT =AA*C*.001*XX
370 XSIG=XSIGT -XP
380 BB=SIG+XSIG
390 CN=8.0+A*(BB**B)
400 XC=CN-C
410 IF(XC.LE.1.0)GO TO 445
415 K=IFIX(XC)+1
420 X = XX/FLOAT (K)
425 GO TO 350
445 C=CN
450 X=X+XX
455 P=P+XP
460 SIG =SIG+XSIG
465 SIGT=SIGT+XSIGT
```

```
470 PRINT 220,X,C,P,SIG, SIGT
495 IF(C.GE.CU)GO TO 510
500 CONTINUE
510 PRINT 515
515 FORMAT (*DO YOU WISH TO CHANGE CONDITIONS?
516CTYPE 1-YES, 0-NO*)
520 INPUT, J
525 IF(J.EQ.1) GO TO 30
540 END
600 SUBROUTINE XPSUBR (C,AJSL, XPX,T,TT,AM)
605 DIMENSION T (50),TT(50)
610 L=IFIX(C)
615 M=L+1
620 XN=1.-4.448*C*0.001/1.9215
622 TC=T(L)+(T(M)-T(L))*(C-FLOAT (L))
625 TTC=TT(L)+(TT(M)-TT(L))*(C-FLOAT(L))
627 AB=8.0*AM*AJSL*TTC/XN
630 D=TC*0.5+0.5*(TC**2.+4.*AB)**0.5
635 XPX=AM*32.*AJSL/(XN*D**2.0)
640 RETURN
650 END
700 ENDPROG
808 .0138.364
809 .0105  0.362
810 .0084   .361
811 .0068   0.360
812 .00530  0.358
813 .004    0.357
814 .003    0.356
815 .0023   0.355
816 .00177  0.353
817 .00139  0.352
818 .00111  0.351
819 .000896 0.349
820 .0007   0.348
821 .00053   .347
822 .00038   .345
823 .00025   .344
824 .00014   .343
825 .00006   .341
826 .00001   .340
827 .00001   .330
828 .00001   .315
829 .00001   .304
830 .00001   .293
831 .00001   .282
832 .00001   .272
833 .00001   .264
834 .00001   .260
835 .00001   .260
836 .00001   .260
837 .00001   .260
838 .00001   .260
839 .00001   .260
840 .00001   .260
```

REFERENCES

References

(1) Konicek, Z., Kos, P.: Gravity Thickening of Industrial Sludges, Paper presented at a Conference of the Czechoslovak Civil Engineering Society (CSVTS) - BRNO 1970.

(2) Bacon, V.W., and Dalton, F.E., Chicago Metropolitan Sanitary District Makes No Little Plans, Public Works New York, Vol. 97, November,1966; p66

(3) Kos, P., "Sludge Gravity Thickening," Ph.D. Thesis-University of Massachusetts/Amherst. 1976

(4) Dell, C.C., Sinah, J.: Experimental Study of the Behavior of Flocculated Clays in Thickening. Transactions (Section C) of the Institution of Mining and Metallurgy. Vol. 75,1966,p. 139-146.

(5) Dell, C.C. Kaynor, M.B.: Channelling in Flocculated Suspensions, Filtration and Separation, July 1968,p. 323-327.

(6) Kalbskopf, K.H.: Studies on the Design of Gravity Thickening, Water Research, Vol. 6, 1972,pp. 499-502

(7) Anderson, A.S., Hanson, A.S.: A Simulation Model of the Gravity Separation of Concentrated Solids, 6th International Conference on Water Pollution Research, 1972.

(8) Raats, P.A.C.: "Development of Equations Describing Transport of Mass and Momentum in Porous Media, with Special Reference to Soils," Ph.D. Thesis, Univ. of Illinois, 1965, Urbana.

(9) Kos, P., "Application of the Theories of Transport Phenomena in Porous Media to the Description of Compressible Sludges during Filtration Processes,: M.S. Project, Umiversity of Massachusetts, Amherst, Massachusetts (Civil Engineering Dept.), August 1971.

(10) Kos, P., and Adrian, D. D., "Transport Phenomena Applied to Sludge Dewatering", Journal of the Environmental Engineering Division, ASCE Vol. 101, No. 336, Proc. Paper 11781, December, 1975, pp. 947-965.